PUSHING AN $(n-1)$-SPHERE IN S^n ALMOST INTO ITS COMPLEMENT

ROBERT J. DAVERMAN

Bing [2] has proved that if Σ is a 2-sphere in the 3-sphere S^3, U is one of its complementary domains, and ϵ is a positive number, then there exist a zero-dimensional subset T of Σ and a map of Σ into $U \cup T$ such that no point is as much as ϵ in distance from its image. In discussing possible generalizations for an $(n - 1)$-sphere Σ in the n-sphere S^n, Wilder [9] conjectured that T should be at most an $(n - 3)$-dimensional subset of Σ. Here we provide a stronger solution to his conjecture. For $n \geq 5$, T can be obtained of dimension at most one. The property fundamental to this work is given in Theorem 2, namely, for a complementary domain U of an $(n - 1)$-sphere Σ in S^n, $n \geq 5$, there exists a one-dimensional set F such that $U \cup F$ is 1-ULC.

1. Definitions and notation. For a positive integer k, I^k denotes a k-cell, ∂I^k its boundary, and Int I^k its interior.

Let S denote a space with a metric ρ. For $A \subset S$ and $\epsilon > 0$, $N_\epsilon(S)$ denotes $\{s \in S \mid \rho(s, A) < \epsilon\}$, diam A denotes the diameter of A, Cl A denotes the closure of A, and Bd A denotes the topological boundary of A in S. For two maps f and g of a compact space X into S, $\rho(f, g)$ is defined as lub $\{\rho(f(x), g(x)) \mid x \in X\}$.

Essential to the development of this paper are the ulc and ULC properties (See [8; 292 ff] and [3].). We define only the uniform properties required here. Let i be a nonnegative integer and S a space with (fixed) metric ρ. We say that S is i-ULC (uniformly locally i-connected) if corresponding to each $\epsilon > 0$ there exists a $\delta > 0$ such that each map of ∂I^{i+1} into a δ-subset of S can be extended to a map of I^{i+1} into an ϵ-subset of S. If S is i-ULC for $i = 0, 1, \cdots, k$, then we say that S is ULC^k. Similarly, for $A \subset S$ we say that A is i-ULC in S if corresponding to each $\epsilon > 0$, there exists a $\delta > 0$ such that each map of ∂I^{i+1} into a δ-subset of A can be extended to a map of I^{i+1} into an ϵ-subset of S, which definition we apply here only in the case $i = 1$. Furthermore, we say that S is i-ulc (uniformly locally homologically i-connected) if to each $\epsilon > 0$, there corresponds a $\delta > 0$ such that each i-cycle (integer coefficients) supported on a δ-subset of S bounds homologically an $(i + 1)$-chain having support in an ϵ-subset of S. Again, S is ulc^k if it is i-ulc for $i = 0, \cdots, k$.

We make applications of several techniques, results, and notations from [5]. In particular dim S denotes the dimension (covering, small inductive, large inductive) of a separable metric space S. Contrary to occasional usage in the

Received May 10, 1972. Revisions received August 10, 1972. The author was supported in part by NSF Grant GP19966.