CONCORDANCE OF DIFFEOMORPHISMS AND THE PASTING CONSTRUCTION

ROBERT WELLS

1. Introduction. If N is a smooth manifold, let Diff (N) be the group of diffeomorphisms of N and let $\mathcal{C}(N)$ be the normal subgroup of those diffeomorphisms concordant to the identity. Following the notation of Antonelli-Burghelea-Kahn, we will write Diff $(N)/\mathcal{C}(N) = \pi_0$ (Diff : N). The object of this paper is to develop some techniques towards the computation of the groups π_0 (Diff : N) for a certain class of manifolds N.

We will be interested primarily in the groups π_0 (Diff : ∂M) for smooth compact connected orientable m-manifolds M having the homotopy type of finite k complexes with $2 \leq k$ and $2k + 3 \leq m$. From now on we assume that M satisfies this condition. The first step will be to approximate the groups π_0 (Diff : M) and π_0 (Diff : ∂M) respectively by the groups π_0 (Diff_0 : M) $=$ π_0 (Diff : $M - \partial M$) and π_0 (Diff_0 : ∂M) $=$ Diff_0 $(\partial M)/\mathcal{C}(\partial M \times R)$, where Diff_0 (∂M) is the group of end-preserving diffeomorphisms $\partial M \times R \to \partial M \times R$. To express the relations among these groups we introduce the two subsets Wh (∂M) and Wh $(\tau(M))$ of the Whitehead group Wh $(\pi_1(M))$ defined by Wh (∂M) is equal to the group of Whitehead torsions of h-cobordisms from ∂M to ∂M and Wh $(\tau(M))$ is equal to the set of Whitehead torsions of homotopy equivalences $f : M \to M$ such that $f^*\tau(M)$ is equivalent to $\tau(M)$. Then we have a commutative diagram (of crossed homomorphisms) with exact rows and columns

$$
\begin{array}{ccccccccc}
 & & \downarrow & & \downarrow & & \downarrow & & \\
1 & \to & \pi_0(\text{Diff} : M) & \to & \pi_0(\text{Diff}_0 : M) & \to & \text{Wh}\,(\tau(M)) & \to & 1 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & \pi_0(\text{Diff} : \partial M) & \to & \pi_0(\text{Diff}_0 : \partial M) & \to & \text{Wh}\,(\partial M) & \to & 1
\end{array}
$$

where the kernel of π_0 (Diff : ∂M) $\to$ π_0 (Diff_0 : ∂M) is a subquotient of Wh $(\pi_1(M))$. Thus we may use this diagram to make reasonable estimates of π_0 (Diff : ∂M) in terms of π_0 (Diff_0 : ∂M); in the case Wh $(\pi_1(M)) = 0$ we have π_0 (Diff : ∂M) $=$ π_0 (Diff_0 : ∂M).

Next we seek to compute the group π_0 (Diff_0 : ∂M). In this direction we begin with a homomorphism $d : \pi_0$ (Diff_0 : ∂M) $\to$ π_0 (Diff_0 : M) such that the following diagram commutes.

Received February 18, 1972. Revisions received July 19, 1972.