CONCERNING COSTABILITY OF COMPACT SEMIGROUPS

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In [6] Lawson constructed a one-dimensional compact semilattice L that does not have small semilattices at its identity. It will be shown that this property of not having small semilattices is equivalent to the property that if $f : S \to L$ is a continuous homomorphism from a compact semigroup S onto L, then the dimension of S is bigger than or equal to the dimension of L. Hence one can look at this concept from the dimensional point of view. (Though not in print, Lawson has alluded to this concept.) This paper also generalizes the concept of dimension costability to general compact semigroups and we will show that a class of semigroups called the cylindrical semigroups is costable.

1. Introduction. The cohomology used in this paper is the Alexander-Wallace-Spanier cohomology with respect to some nontrivial abelian group. The dimension function is also the cohomological definition of codimension [3]. The following is a topological theorem.

THEOREM 1.1. *Let $f : X \to Y$ be a continuous function from a compact Hausdorff space X to another such space Y. Then*

$$\mathrm{cd}\, X \leq \mathrm{ind}\, Y + \mathrm{cd}\, f,$$

where cd *is codimension and* ind *is inductive dimension and*

$$\mathrm{cd}\, f = \max\{\mathrm{cd}\, f^{-1}(p) \mid p \in Y\}.$$

Proof. If $\mathrm{ind}\, Y = -1$, then the theorem is vacuously satisfied since Y is assumed to be nonempty. Let $\mathrm{ind}\, Y = n \geq 0$. Induct on n. Let $\mathrm{cd}\, f = m$, and one will show $\mathrm{cd}\, X \leq n + m$.

Suppose $\mathrm{cd}\, X \not\leq n + m$. Then there exists $e \neq 0$, $e \in H^{n+m+1}(X, A)$ for some closed subset A. By the Hausdorff Maximality Principle we can find a closed subset F of X such that $e|_{(F, F\cap A)} \neq 0$ and if K is a proper closed subset of F, then $e|_{(K, K\cap A)} = 0$. Let $h = e|_{(F, F\cap A)}$.

We can see that $f(F)$ is not a point; otherwise $\mathrm{cd}\, f \neq m$. Since $\mathrm{ind}\, Y = n$, then $\mathrm{ind}\, f(F) \leq n$, and it follows from the definition that there exist closed subsets P and Q of Y such that $f(F) = P \cup Q$ and $\mathrm{ind}\,(P \cap Q) \leq n - 1$. Let $R = F \cap f^{-1}(P)$ and $S = F \cap f^{-1}(Q)$.

Note $S \cap R = F \cap f^{-1}(P \cap Q)$. Hence by induction $\mathrm{cd}\,(S \cap R) \leq n + m - 1$.

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