GENERALIZED FRIEZE PATTERNS

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1. Introduction. Following Coxeter [1], we define a frieze pattern based on the positive real numbers $c_{11}, c_{21}, \cdots, c_{n1}$ as an array of the form

$$
(1.1)\qquad
\begin{array}{llll}
0 & & & \\
1 & 0 & & \\
c_{11} & 1 & 0 & \\
c_{21} & c_{22} & 1 & \\
\vdots & \vdots & \vdots & \vdots \\
c_{n1} & c_{n2} & c_{n3} & \\
1 & c_{n+1,2} & c_{n+1,3} & \\
0 & 1 & c_{n+2,3} & \\
 & 0 & 1 & \\
 & & 0 &
\end{array}
$$

where the elements of the j-th column are defined successively by the relations

$$
(1.2)\qquad c_{ij}c_{i-1,j-1} - c_{i,j-1}c_{i-1,j} = 1.
$$

Coxeter proved that the columns in a frieze pattern are periodic of period $n + 3$, that is, $c_{r1} = c_{r+n+3,n+4}$. He also found a simple criterion so that the elements of the frieze pattern are positive integers. Namely, it is necessary and sufficient that the first column consist of positive integers which satisfy the condition

$$
(1.3)\qquad c_{r1} \mid (c_{r-1,1} + c_{r+1,1}).
$$

He also discusses the geometric significance and the history of frieze patterns.

Here we are concerned with a generalization of the frieze pattern. Let m and n be positive integers with $m \geq 2$ and let $c_{11}, c_{21}, \cdots, c_{n1}$ be positive real numbers. Define the array (1.1) according to the following requirements. If $2 \leq k \leq m$, then

$$
(1.4)\qquad \det \begin{bmatrix} c_{r+1-k,1} & \cdots & c_{r+1-k,k} \\ \vdots & & \vdots \\ c_{r1} & \cdots & c_{rk} \end{bmatrix} = 1
$$

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