ORIENTABILITY OF BUNDLES

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1. Introduction. The object of this note is to explore the conditions under which a vector bundle is orientable for a generalized cohomology theory defined by a ring spectrum. We also discuss the question of how many orientations there are. The problem is reduced to the existence and enumeration of cross-sections of a fibration associated with the original bundle. We call this fibration the *determinant fibration* by analogy with the determinant bundle that arises in connection with ordinary orientability.

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2. Classifying orientations. Let $p : E \rightarrow B$ be a real vector bundle of dimension n over a connected CW-complex B. We assume that either B is finite or the bundle is numerable. We will let j be the inclusion of a fiber. If h is a generalized cohomology theory, then contained in $h^n(R^n, R^n - 0)$ is the subset G of the suspensions of the units in h^0 (point). The bundle is said to be *orientable* for h if there exists an element $U \in h^n(E, E^0)$ such that $j^*U \in G$. U is referred to as an *orientation* or a *Thom class*. In this case we have the Dold-Thom isomorphism theorem.

THEOREM 1. [2; 41]. *If E is orientable with orientation U, then U generates $h(E, E^0)$ as a free $h(B)$-module.*

We wish to classify orientations of E under the assumption that one exists. Our first observation is the following. There is a one-to-one correspondence between orientations and units in $h^0(B)$. For if t is a unit and U is an orientation, then $j^*(tU) \in G$ and so tU is an orientation. If U and V are orientations, then since each generates, $V = tU$ for some t and $U = t'V$ for some t' and clearly $tt' = t't = 1$.

COROLLARY 2. *Let B be a connected complex and let $\rho_x : \{x\} \rightarrow B$ be the inclusion of a point. Then the units of $h^0(B)$ are the inverse images under ρ_x^* of the units in h^0 (point).*

Proof. Take $n = 0$.

Now we make the assumption that h is determined as in [5] by a spectrum $\mathfrak{A}$ which in fact is a *loop spectrum.* This is a sequence of basepointed spaces $\mathfrak{A}_i$ and homotopy equivalences $\epsilon_i : \mathfrak{A}_i \rightarrow \Omega\mathfrak{A}_{i+1}$. Then $\tilde{h}^i(X) = [X, \mathfrak{A}_i]$ if X has a

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