SPACES DEFINED BY SEQUENCES OF OPEN COVERS
WHICH GUARANTEE THAT CERTAIN SEQUENCES
HAVE CLUSTER POINTS

R. E. HODEL

1. Introduction. In recent years much of the research in point set topology
has been devoted to a study of certain generalizations of metrizable spaces.
In particular I have in mind developable spaces, $\omega$ spaces, semi-stratifiable
spaces, first countable and $q$-spaces, Nagata spaces, $\sigma$ and $\Sigma$ spaces, $wM$ spaces
and others. Each of these classes of spaces can be characterized (or is actually
defined) in terms of a sequence of covers which guarantee that certain sequences
have cluster points. Such characterizations give a unified approach to these
generalized metrizable spaces and suggest problems and new classes of spaces.
(See [15] and [14; §5].)

In this paper we study some new classes of generalized metrizable spaces
which are defined by a sequence of open covers which guarantee that certain
sequences have cluster points. In §3 we introduce the class of $wN$-spaces as
a generalization of Nagata spaces and prove that every Hausdorff developable
$wN$-space is metrizable. In §4 we introduce some new classes of first countable
and $q$-spaces and give conditions under which these spaces are metrizable.
In §5 we study the relationship between the $wM$-spaces of Ishii and the spaces
introduced in §§3 and 4. Finally in §6 we give some applications of our results
to the problem of metrizability.

2. Preliminaries. We begin with some definitions and known results which
will be used throughout this paper. Unless otherwise stated no separation
axioms are assumed; however regular spaces are always $T_1$. The set of natural
numbers will be denoted by $\mathbb{N}$ and $i$, $j$, $k$ and $n$ will denote elements of $\mathbb{N}$.

A space $X$ is developable if there is a sequence $\mathcal{G}_1$, $\mathcal{G}_2$, $\cdots$ of open covers
of $X$ such that for each $x$ in $X$ \{st $(x, \mathcal{G}_n)$: $n$ in $\mathbb{N}$\} is a fundamental system of
neighborhoods of $x$. A regular developable space is called a Moore space.
Bing [4] proved that every collectionwise normal Moore space is metrizable.

Let $X$ be a space, let $\mathcal{G}_1$, $\mathcal{G}_2$, $\cdots$ be a sequence of covers of $X$, and consider
the following conditions on $\mathcal{G}_1$, $\mathcal{G}_2$, $\cdots$.

1. If $x_n \in \text{st}^\pm (p, \mathcal{G}_n)$ for $n = 1, 2, \cdots$, then $\langle x_n \rangle$ has a cluster point.

2. If $x_n \in \text{st} (p, \mathcal{G}_n)$ for $n = 1, 2, \cdots$, then $\langle x_n \rangle$ has a cluster point.

3. If $x$ and $y$ are distinct points of $X$, then there exists $n$ in $\mathbb{N}$ such that
$y \notin \text{st}^\pm (x, \mathcal{G}_n)$.

4. If $x$ and $y$ are distinct points of $X$, then there exists $n$ in $\mathbb{N}$ such that
$y \notin \text{st}^\pm (x, \mathcal{G}_n^-)$.

Received December 31, 1971.

253