A REMARK ON FINITE DIMENSIONAL COMPACTIFICATIONS

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The Bohr compactification of a (discrete) group G is defined as a pair $(\sigma, \hat{G})$, where $\hat{G}$ is a compact group and σ is a dense representation of G into $\hat{G}$, with the property that any diagram

$$\begin{array}{ccc} \hat{G} & & \\ {\scriptstyle\sigma}\uparrow & & \\ G & \underset{\xi}{\rightarrow} & B \end{array}$$

with ξ a dense representation into the compact group B will complete to

$$\begin{array}{ccc} \hat{G} & & \\ {\scriptstyle\sigma}\uparrow & \searrow^{\hat{\xi}} & \\ G & \underset{\xi}{\rightarrow} & B \end{array}$$

with $\hat{\xi}$ an onto homomorphism.

For a number of standard examples the dimension of $\hat{G}$ is infinite. This is the case, for example, for the integers, the rationals, free groups and so forth. And indeed we note the following

THEOREM. *The compact connected finite dimensional groups which can serve as Bohr compactifications (of discrete groups) are precisely the compact connected semi-simple Lie groups.*

We require first the following

LEMMA. *Let A be a discrete abelian group. Then $\hat{A}$ is zero dimensional if and only if A is the direct sum of finite cyclic groups where the set of orders is bounded. If $\hat{A}$ is finite dimensional, then it is zero dimensional.*

It is known that $\hat{A}$ may be viewed as the character group of the discrete dual. That is to say,

$$\hat{A} = ch\{(chA)_d\}.$$

Now $\hat{A}$ is zero dimensional if and only if $(chA)_d$ is a torsion group. Now a torsion group having a compact topology is topologically isomorphic with the cartesian product of cyclic groups of bounded orders, [3]. Thus, chA, being a compact torsion group,

$$ch\ A = \times_{\alpha} F_{\alpha}\ .$$

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