EXTENDING MONOTONE DECOMPOSITIONS OF 2-SPHERES TO TRIVIAL DECOMPOSITIONS OF E^3

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1. Introduction. In recent years considerable interest has been given to the problem of modifying certain upper semi-continuous decompositions of E^3 so that the new decomposition is in some way simpler. Generally this modification has taken the form of adding non-degenerate elements and the simplification sought is that the new decomposition yield E^3 as its decomposition space. (See [1], [2], [3] and [7].)

In this paper we start with a monotone decomposition G of a 2-sphere embedded as the unit ball in E^3 and modify G by enlarging each non-degenerate element in such a way that the new decomposition has E^3 for its decomposition space and can be realized by a pseudo-isotopy of E^3 onto itself.

Bing has shown [4] that if we have a monotone decomposition G of the 2-sphere and enlarge it to a decomposition of the 3-ball by adding each point of the interior of the ball to G, then the decomposition space will be the closure of one complementary domain of some cactoid embedded in S^3. This theorem is proven using a characterization of S^3 and gives no information about the embedding of the cactoid. Here we do Bing's theorem in a more geometric way and answer questions raised by him in [4].

In §2 of this paper we do the main theorem in a very special case. This is done to illustrate the technique. In §3 we prove an embedding theorem for cactoids, which is of independent interest, then use it in the proof of the general theorem, §5.

We shall use the techniques developed here in another paper where our desire is to extend a decomposition defined on the image of a 2-sphere under a special sort of map. In that paper we will answer questions raised by Bing concerning his monotone map of E^3 onto itself which is not compact.

2. Definitions and notation. The 2-sphere S^2 we work with is the unit ball in E^3 and B^3 denotes the closure of its bounded complementary domain. G is a monotone upper semi-continuous decomposition of S^2 (standard definitions) and $\pi: S^2 \rightarrow S^2/G$ denotes the natural projection from S^2 to the decomposition space S^2/G. For simplicity, if we have a copy of S^2/G sitting in E^3 we still denote by π the map from S^2 to this copy. Viewed in this light, we are extending maps from $S^2 \subset E^3$ into E^3 to take E^3 onto itself.

A cactoid is a locally connected continuum in which each true cyclic element is a 2-sphere. By [5] we know that these are precisely the monotone images of 2-spheres. We use cyclic element theory given in [6].

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