HOMOMORPHISMS ON CONNECTED TOPOLOGICAL LATTICES

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In this paper we deal with several aspects of the theory of homomorphisms on connected topological lattices of finite breadth. Suppose that L is such a lattice. In the first section we show that if φ is a homomorphism of L onto a locally compact, connected, distributive lattice, then φ must also be continuous. Open homomorphisms occupy our attention in the second section. If L is distributive with breadth n and φ is an open homomorphism, we are able to show that φ must be an iseomorphism if the range space is a locally compact, connected topological lattice which is locally of breadth n. In the last section we show that if L is distributive, then it has enough continuous homomorphisms onto I to separate points and closed ideals and to separate points and closed dual ideals.

0. Preliminaries and definitions. Suppose that L is a lattice with operations $\vee$ and $\wedge$. L is a topological lattice if L is a Hausdorff space and $\vee$ and $\wedge$ are continuous. The set of homomorphisms of L onto a lattice M is denoted by hom (L, M). If L and M are topological lattices, the set of continuous homomorphisms of L onto M is denoted by **Hom** (L, M). By the interval from a to b, written $[a, b]$, we shall mean $\{x \in L; a \leq x \leq b\}$. $[a, b)$ and $(a, b]$ have the obvious meanings. A chain is a linearly ordered topological lattice with the interval topology. $a \in L$ is meet-irreducible if $a = x \wedge y$ implies that $a \in \{x, y\}$. A subset A of L is meet-redundant if there is a proper subset B of A such that $\wedge B = \wedge A$. A is said to be meet-irredundant if it is not meet-redundant. The previous definitions have dual formulations for join. We say that breadth of L is n, written Br $(L) = n$, if L has a meet-irredundant subset of n elements but no meet-irredundant subset containing $n + 1$ elements. L is locally of breadth n if every non-empty open subset of L has a meet-irredundant (in L) subset of n elements but does not contain a meet-irredundant set containing $n + 1$ elements. It is readily apparent that if L is locally of breadth n then it is of breadth n while the converse does not hold. A subset A of a topological lattice L is convex if $A = (A \wedge L) \cap (A \vee L)$ and L is said to be locally convex if the topology for L has a base of convex sets. By I we shall mean the real interval [0, 1] with its usual lattice structure. An iseomorphism is an isomorphism which is also a homeomorphism. Finally, by A^* we shall mean the topological closure of the set A.

1. Homomorphisms of connected topological lattices of finite breadth. We shall first show that every homomorphism of a connected topological lattice of

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