LOCAL TRIVIALITY OF COMPLETELY REGULAR MAPPINGS

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1. Introduction. In [2], Dyer and Hamstrom and in [6], Hamstrom, have shown that a completely regular map $p:E \to B$ is a locally trivial fiber map if E is a complete metric space and B is a finite (covering) dimensional metric space and all fibers are homeomorphic to a $k(\leq 3)$-dimensional compact manifold M. In [5], Hall extended this result to the case when M is a non-compact $k(\leq 3)$-dimensional manifold.

The difficulty in extending the above results to the case when the fiber is a higher dimensional manifold M was in the lack of information about the group of homeomorphisms of M. However, the work of Cernavskii [1], a recent work of Edwards and Kirby [4] and a paper of McCarty [8] have provided enough knowledge to extend the results to the case when M is a finite dimensional compact manifold or a Euclidean n-space. In the Euclidean n-space case, we need a technical condition on a metric on the space E.

2. Definitions and statements of theorems. A *completely regular map* p from a metric space (E, ρ), where ρ is a metric on E, onto a metric space (B, d) is a map such that for each $b \in B$ and $\epsilon > 0$ there exists a $\delta(b, \epsilon) > 0$ such that if $c \in B$ with $d(b, c) < \delta$, then there exists a homeomorphism h of $p^{-1}(b)$ onto $p^{-1}(c)$ such that $\rho(x, h(x)) < \epsilon$ for each $x \in p^{-1}(b)$. The space B will always be assumed to be connected. Then all of the sets $p^{-1}(b)$ are homeomorphic.

A map f of a metric space (X, d_1) onto a metric space (Y, d_2) is said to be a *uniform isomorphism* if f is one-to-one and both f and f^{-1} are uniformly continuous. If there is a uniform isomorphism from X onto Y, then we say (X, d_1) and (Y, d_2) are *uniformly isomorphic.*

With this terminology established, we now state our theorems.

THEOREM 1. *Let $p:E \to B$ be a completely regular map from a bounded complete metric space (E, ρ) onto a finite covering dimensional metric space (B, d). Suppose all fibers are uniformly isomorphic and they are uniformly isomorphic to (R^n, σ), where R^n is a Euclidean n-space sitting in a n-sphere (S^n, σ) as $S^n - x$ with a metric σ. Then $p:E \to B$ is a locally trivial fiber map.*

THEOREM 2. (Extension of Dyer and Hamstrom's Theorem). *Let $p:E \to B$ be a completely regular map from a (bounded) complete metric space (E, ρ) onto a finite covering dimensional metric space (B, d). Suppose the fiber is homeomorphic to a compact n-dimensional manifold M. Then $p:E \to B$ is a locally trivial fiber map.*

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