ONE-PARAMETER GROUPS OF FORMAL POWER SERIES

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The problem of analytic iteration led to the study of groups of formal power series, having the form

$$F(z, s) = \sum_{q=1}^{\infty} f_q(s) z^q$$

where the coefficients $f_q(s)$ are analytic functions of the complex parameter s, such that for any two complex numbers s and t the formal law of composition

$$F[F(z, s), t] = F[z, s + t]$$

is valid. E. Jabotinsky [7] gave the explicit form of the coefficients $f_q(s)$ in such a group, in the particular case when $f_1(s) \equiv 1$. P. Erdös and E. Jabotinsky in [4] and I. N. Baker in [1] characterized in this particular case the set of the parameters s for which the series $F(z, s)$ has a non-zero radius of convergence.

Groups of the same form, with $f_1(s) \not\equiv 1$ could not be treated directly by the same methods, because the form of the coefficients $f_q(s)$ becomes very complicated (for the first coefficients, see [6]).

The purpose of the present paper is to investigate the groups with $f_1(s) \not\equiv 1$, applying an appropriate transformation to the parameter s. Part 1 gives the basic definitions and concepts connected with the problem. Part 2 gives the explicit form of the coefficients of such a group, while Part 3 characterizes the set of the parameters for which the power series representing the group has non-zero radius of convergence.

1. Introduction and definitions.

1.1. Let Σ^F denote the linear algebra of the formal power series over the field of complex numbers, having the form

$$F(z) = \sum_{q=0}^{\infty} f_q z^q. \tag{1}$$

(Operations on formal power series are defined in [2, Chapter 1].) Σ^F is equipped with the metric

$$\rho(F, G) = \sum_{q=0}^{\infty} 2^{-q} \frac{|f_q - g_q|}{1 + |f_q - g_q|} \tag{2}$$

for $F, G \in \Sigma^F$.

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