AN ASYMPTOTIC FORMULA IN PARTITION THEORY

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1. Introduction. In 1956 Meinardus [1] gave an asymptotic relation for the logarithm of the number of partitions of totally positive lattice points. A point of n-dimensional space with a given cartesian coordinate system is called "totally positive" if all its coordinates are positive.

In this paper we study a more complicated type of problem. We consider a lattice G and a sublattice G' together with lattice points $\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_l$ of G with the property that $\mathbf{r}_i \not\equiv \mathbf{r}_j \pmod{G'}$ if $i \neq j$, and that every point $\mathbf{g}$ of G is congruent modulo G' to some vector $c_1\mathbf{r}_1 + c_2\mathbf{r}_2 + \cdots + c_l\mathbf{r}_l$, where the coefficients c_i are integers. Let $P(\mathbf{g})$ be the number of partitions of $\mathbf{g}$ into totally positive summands which lie in G, and are congruent to some $\mathbf{r}_i$ modulo G'. Two partitions are regarded identical if they differ only in the order of their summands. Our main result is an asymptotic formula for $\log P(\mathbf{g})$. The author wishes to express her sincere gratitude to Professor Wolfgang M. Schmidt for his guidance, encouragement and his many helpful suggestions.

Let $0 < c_0 < c_1$ be arbitrary but fixed. We say a totally positive vector $\mathbf{m}$ is "reduced" if

$$c_0(N\mathbf{m}/^{1/n} < m_\nu < c_1(N\mathbf{m})^{1/n} \qquad (\nu = 1, 2, \cdots, n),$$

where

$$\mathbf{m} = (m_1, m_2, \cdots, m_n),$$

and

$$N\mathbf{m} = \prod_{i=1}^{n} m_i .$$

For reduced $\mathbf{g}$, we write $\mathbf{g} \Rightarrow \infty$ when each of its components g_i tends to infinity.

THEOREM. *For* $\mathbf{g} \Rightarrow \infty$

$$\log P(\mathbf{g}) = (n+1)\left\{\frac{l\zeta(n+1)}{\det G'} N\mathbf{g}\right\}^{1/n+1} \qquad (1 + o(1)).$$

Our method of approach is essentially Meinardus'. However, in contrast to Meinardus we first reduce the problem to the special case where our lattices are integer lattices, and where, in fact, G' is a multiple of the fundamental lattice. This reduction makes it much easier to apply a certain Tauberian theorem proved by Meinardus. It is no longer necessary to prove a monotonicity property of the partition function. The proof of such a property in general is

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