CENTRAL IDEMPOTENT MEASURES ON SIN GROUPS

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1. Introduction. Let G be a locally compact group. $M(G)$, the space of regular finite measures on G, forms a Banach algebra where the multiplication is given by convolution:

$$\mu * \nu(E) = \int \mu(Ex^{-1})\, d\nu(x).$$

A measure μ is *idempotent* if $\mu * \mu = \mu$; it is *central* if it lies in the center of $M(G)$. The idempotent measures for abelian groups have been classified by Cohen [1]. The central idempotent measures for certain compact groups, including the unitary groups, have been characterized in [5].

If G is abelian, then an idempotent measure is supported on a compact subgroup (cf. [7, Theorem 3.3.2]). That this is false in general has been demonstrated by Rudin in [6] where an example is given of an idempotent on a discrete group that is not supported on a finite subgroup. However, it has been shown by the author [4] that in the discrete case a *central* idempotent is supported on a compact (i.e. finite) subgroup. It is the purpose of this paper to extend this to SIN groups.

$G \in$ [SIN] provided every neighborhood of the identity in G contains a neighborhood of the identity which is invariant under all inner automorphisms. Such groups have been studied by Grosser and Moskowitz [2]; see in particular their Theorem 4.2. The *support group* of a measure $\mu \in M(G)$ is the smallest closed subgroup of G which supports μ.

THEOREM 1. *Let* $G \in$ [SIN] *and* μ *be a central idempotent measure on* G. *Then the support group of* μ *is compact.*

Theorem 1 is proved in §3. In §2 we prove some results about groups which support central idempotents without requiring that they be in [SIN]. It seems reasonable to conjecture that Theorem 1 is true without the assumption that $G \in$ [SIN]. This requires showing that if G is the support group of a central idempotent then $G \in$ [SIN]. However, the proof given here uses the assumption several times (in the proofs of Lemmas 4, 5 and 6 of §3).

2. Groups supporting central idempotent measures. G' will denote the closure of the commutator subgroup of the locally compact group G. For $x \in G$, $C(x)$ is the conjugacy class containing x. [*FC*] is the class of groups for

Received April 21, 1969. This research was supported in part by the National Science Foundation.