STRONGLY CONVERGENT DERIVATION AUTOMORPHISMS ON A CLASS OF WILDLY RAMIFIED v-RINGS

By Martin Heinzer

I. Introduction. Let R_e be a ramified v-ring with ramification e. That is, R_e is a complete, discrete, rank one, valuation ring having characteristic zero with residue field k of characteristic $p(p \text{ prime} \neq 2)$ and pR_e is the e-th power of the maximal ideal M of R_e. If $D = \{D_i\}$ is a higher derivation on R_e, it is convergent if for each $a \in R_e$ the series $\sum_{i=0}^{\infty} D_i(a)$ converges in the topology generated by the maximal ideal M. If D is convergent and $\alpha_D(a) = \sum D_i(a)$ for each $a \in R_e$, then α_D is an automorphism of R_e. The set of derivation automorphisms α_D form an invariant subgroup G_D of the automorphism group G of R_e. If $D_i(R_e) \subset M^i$ for each i, α_D is a strongly convergent derivation automorphism. The set G_S of all such α_D is also an invariant subgroup of the automorphism group G. [2; 34, Theorem 3].

Let ϵ be the identity map and for each $i > 0$ let $G_i = \{\alpha \mid \alpha \in G \text{ and } \alpha = \epsilon, \bmod M^i\}$, and $H_i = \{\alpha \mid \alpha \in G_i \text{ and } \alpha(m) - m \in M^{i+1} \text{ for each } m \in M\}$. The series

1.1 $$G_1 \supset H_1 \supset G_2 \supset H_2 \supset \cdots$$

is the ramification series of R_e.

MacLane [5] has evaluated the factors of 1.1 in the case $e = 1$. More recently, using derivation automorphisms, the factors of 1.1 have been evaluated for a larger class of complete local rings by Heerema [1] [3] and Neggers [6]. If R is an unramified complete regular local ring, $G_D = G_S = H_1$ [1]. If R_e is tamely ramified, then again $G_D = G_S = H_1$. Also, G_1 is a semi-direct product of $G(R_e, R)$ and H_1 where R is an unramified v-subring of R_e, $[R_e : R] = e$, and $G(R_e, R)$ is the group of automorphisms of R_e/R. A similar relationship exists between $G(R_e, R)$, G_D, and G_1 $(= H_1)$ in the simplest wildly ramified case, when $e = p$ [3].

The object of this paper is to determine the relationships between G_S, G_D, and the ramification series 1.1 in the case $e = p$. It is shown that two parameters, expo R_p and res R_p, together with the condition $G_2 = H_2$ completely determine these relationships. If π is a prime element and $\pi^p + pu = 0$ with $\bar{u} \notin k^p$($\bar{u}$ is the residue of u), then expo $R_p = 0$ and res $R_p = \bar{u}$. Otherwise π may be chosen so that $\pi^p + p(1 + \pi^t v) = 0$ and the parameters depend on t and $\bar{v}$ (see 2.8 and 2.9 for precise definitions).

The main results of this study are stated in Theorems 4.11, 4.13, 4.14 and

Received January 11, 1969. This research was supported in part by NSF Grant GP 8424. It consists of Excerpts from the Author's Doctoral Dissertation, Florida State University.