SEMINORMAL AND C-COMPACT SPACES

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Introduction. In a previous paper [7], the author has defined a space (X, τ) to be C-compact if given a closed subset Q of X and a τ-open cover $\mathcal{O}$ of Q, then there exists a finite number of elements in $\mathcal{O}$, say 0_i, $1 \leq i \leq n$, with $Q \subset \bigcup_{i=1}^{n} \bar{0}_i$. Every continuous function from a C-compact space into a Hausdorff space is closed and there exist C-compact Hausdorff spaces which are not compact [7].

In § one an example is given showing that the product of C-compact spaces need not be C-compact, thus resolving one of the questions posed in [7]. The property "seminormal" is introduced in § two, and some relations between this concept and that of C-compactness are examined in § 3.

All spaces in this paper are assumed to be Hausdorff.

I. Product of C-compact spaces. The example in this section shows that even the product of a C-compact space with the closed unit interval need not be C-compact. However, if a product is C-compact, then each factor is C-compact.

Theorem 1. *If $X = \prod_\alpha X_\alpha$, where no X_α is empty, is C-compact, then each X_α is C-compact.*

Proof. A direct consequence of the easily established fact that the continuous image of a C-compact space is C-compact.

Example 1. Let Z represent the set of positive integers. Let Y denote the subset of the plane consisting of all points of the form $(1/n, 1/m)$ and the points of the form $(1/n, 0)$ for n and m in Z. Let $X = Y \cup \{\infty\}$. Topologize X as follows: Let each point of the form $(1/n, 1/m)$ be open. Partition Z into infinitely many infinite equivalence classes, $\{Z_i\}_{i=1}^{\infty}$. Let a neighborhood system for the point $(1/i, 0)$ be composed of all sets of the form $0 \cup F$, where

$$0 = \left\{\left(\frac{1}{i}, 0\right)\right\} \cup \left\{\left(\frac{1}{i}, \frac{1}{m}\right) \,\middle|\, m \geq k\right\}$$

and

$$F = \left\{\left(\frac{1}{n}, \frac{1}{m}\right) \,\middle|\, m \in Z_i \quad \text{and} \quad n \geq k\right\}$$

for some $k \in Z$. Let a neighborhood system for the point ∞ be composed of all sets of the form $X \backslash T$ where

$$T = \left\{\left(\frac{1}{n}, 0\right) \,\middle|\, n \in Z\right\} \cup \bigcup_{i=1}^{k} \left\{\left(\frac{1}{i}, \frac{1}{m}\right) \,\middle|\, m \in Z\right\} \cup \left\{\left(\frac{1}{n}, \frac{1}{m}\right) \,\middle|\, m \in Z_i, n \in Z\right\}$$

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