ACTIONS WITH TOPOLOGICALLY RESTRICTED STATE SPACES II

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If T is a topological semigroup and X is a Hausdorff space, an Act (act) is a continuous function μ from $T \times X$ onto (into) X such that $\mu[s, \mu(t, x)] = \mu(st, x)$ for each $s, t \in T$ and $x \in X$. X is called the state-space, and we say that T Acts (acts) on X. If $S \subset T$ and $A \subset X$, we write SA rather than $\mu(S, A)$ when no confusion is likely. We say that T acts effectively on X if $s = t$ (in T) whenever $sx = tx$ for each $x \in X$. A compact connected semigroup with identity is called a clan. We consider only Hausdorff topological spaces and our dimension function is the one defined by Cohen [3].

In §1 we show that if X is a compact one-dimensional space, T is a compact connected semigroup which is normal or has an identity, and T Acts on X with some element acting as a constant mapping, then X is a generalized tree. Moreover, a metric generalized tree admits a thread Action with the zero acting as a constant map and thus is contractible. In §2 we consider the class of spaces which admit no effective non-group clan Action. We show that metric n-indecomposable continua, compact manifolds and group supporting spaces, and certain homogeneous continua belong to this class.

A semigroup T is called a thread if its underlying space is an arc with one endpoint an identity for T and the other a zero for T. We use $E(S)$ to denote the set of idempotents in a semigroup S and $K(S)$ to denote its minimal ideal (when this exists). A semigroup T is called normal iff $Tt = tT$ for each $t \in T$. The boundary, interior and closure of a set A are denoted by $F(A)$, A^0 and A^* respectively. We use the Alexander–Kolomogoroff–Spanier–Wallace cohomology with an arbitrary abelian coefficient group (unless otherwise specified).

Let the semigroup T Act on the space X. In [10] we defined a "natural" quasi-order on X by $x \leq y$ iff $x \in y \cup Ty$. We also consider the associated equivalence relation δ on X ($x\delta y$ iff $x \leq y$ and $y \leq x$) and let $\delta[x]$ denote the equivalence class containing x. The natural quasi-order is closed in $X \times X$ when T is compact.

Assume now that T is compact. Let $\mathcal{E} = \{(s, t) \in T \times T \mid sx = tx \text{ for each } x \in X\}$. Then $\mathcal{E}$ is a closed congruence on T, and we let ρ denote the natural surmorphism of T on $T/\mathcal{E}$. Define $\nu : T/\mathcal{E} \times X \to X$ by $\nu(\rho(t), x) = tx$. Then $T/\mathcal{E}$ Acts effectively on X via ν.

Let I be a closed ideal of T and let q denote the natural surmorphism of T on the Rees–Quotient semigroup T/I. Assume now that X is compact and let Δ denote the diagonal. Then $(IX \times IX) \cup \Delta$ is a closed equivalence on X, and we let r denote the natural projection of X on the decomposition space

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