ASYMPTOTIC PERIODIC POINT THEOREMS

BY BENJAMIN HALPERN

Introduction. Let X be a Hausdorff space, S a subset of X, and f a continuous map from S into X. By an asymptotic periodic point theorem for the map f we mean one in which the existence and properties of periodic points of f with "small" periods are deduced from hypotheses on the structure of the iterates f^n of f, especially for "large" integers n.

In a paper by the writer [7], several theorems on periodic points were established which stemmed from an analysis of the Lefschetz numbers of the iterates of f. It is the intent of this paper to extend these results so as to establish asymptotic periodic point theorems generalizing the asymptotic fixed point theorems of Browder [1].

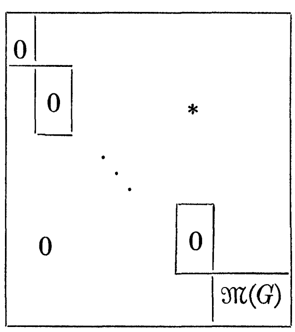
1. Most of the theorems of [7] are stated in terms of the dimension β_n of the spaces $H_n(X)$ where H is a homology theory with rational coefficients. In order to carry over these results efficiently to asymptotic periodic point theorems we will show how a hypothesis on the β_n's can be replaced by the same hypothesis on a sequence of integers B_n such that rank $((f_n^*)^{\dim H^n(X)}) \leq B_n$ where $f : X \to X$ is the map in question and $f_n^* : H^n(X) \to H^n(X)$ is the induced Cěch cohomology homomorphism.

First consider a finite dimensional vector space V and a linear map $L : V \to V$. Suppose W is an invariant subspace, $L(W) \subset W$, and m an integer such that $L^m(V) \subset W$. Let $F = L \mid W : W \to W$. There is a close relation between $p(L)$ and $p(F)$, the characteristic polynomials of L and F. In fact, if $M = \dim W$ and $N = \dim V$, then $p(L)(\lambda) = \lambda^{N-M}p(F)(\lambda)$. To see this, consider the sequence of subspaces

$$V \supset L(V) \supset L^2(V) \supset \cdots \supset L^m(V).$$

We may assume $L^{m+1}(V) = L^m(V)$.

Choose a basis $v_1 \cdots v_{n_0} \cdots v_{n_1} \cdots \cdots v_{n_m}$ for V such that $v_1 \cdots v_{n_p}$ is a basis for $L^{m-p}(V)$. Set $Q = L^m(V)$ and $G = L \mid Q = F \mid Q$. The matrix $\mathfrak{M}(L)$ of L with respect to $v_1 \cdots v_{n_m}$ is of the form



Received October 31, 1968; in revised form September 21, 1970.