NONLINEAR, ANALYTIC PARTIAL DIFFERENTIAL EQUATIONS WITH GENERALIZED GOURSAT DATA

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1. Introduction. In this paper we shall extend results previously obtained by one of us in collaboration with A. K. Aziz and H. C. Howard [1] concerning the existence and uniqueness of solutions to a class of nonlinear partial differential equations with "generalized Goursat data" and at the same time simplify the analysis. In [1], the nonlinear partial differential equation

$$\Delta u = \tilde{f}(x, y, u, u_x, u_y) \tag{1}$$

was considered with regard to solutions satisfying the generalized Goursat data

$$u_x(x, y) = a_0(x)u(x, y) + a_1(x)u_y(x, y) + k(x) \tag{2}$$

on the analytic curve $\Gamma_1' \equiv \{(x, y) \mid y = f_1(x), 0 \leq x \leq a\}$ and

$$u_y(x, y) = b_0(y)u(x, y) + b_1(y)u_x(x, y) + l(y) \tag{3}$$

on the analytic curve $\Gamma_2' \equiv \{(x, y) \mid x = f_2(y), 0 \leq y \leq b\}$ with $u(0, 0) = \gamma$, $\Gamma_1' \cap \Gamma_2' = (0, 0)$.

In [1] the *complex* analytic properties of the initial value problem were not taken full advantage of. Indeed, because of this, existence and uniqueness could not be proved for the family of carriers whose analytic continuation to surfaces in the space of two complex variables were not described by an injective mapping; in particular, carriers consisting of two non-co-linear rays from the origin were excluded. In the present study we remove these restrictions, and in fact, show by a conformal mapping that this class of carriers is actually of central importance. The result is not only a significant extension of previous work but a new and more elegant analysis of the whole problem. In what follows we treat in detail the special case when

$$a_1(x) = b_1(y) = i$$

and then outline in §6 how our results can be modified to handle the more general case. We restrict ourselves to describing in detail this particular case since it retains the salient features of the more general problem and also allows us to provide a purely *analytic* procedure for obtaining the solution.

2. The hyperbolic equation $U_{zz^*} = f(z, z^*, U, U_z, U_{z^*})$. Following the approach used in [1] we consider the situation which arises when x and y are replaced by the two *independent* complex variables $z = x + iy$, and $z^* = x - iy$.

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