GENERALIZED SHEFFER POLYNOMIALS

By W. A. Al-Salam and A. Verma

1. Introduction. Let $\{P_n(x)\}$ be a polynomial set (p.s.), i.e., a sequence of polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$, $\cdots$ for which the degree of $P_n(x)$ is exactly n. A polynomial set is said to be Appell if $P_n'(x) = P_{n-1}(x)$ for all $n \geq 1$. A well known characteristic property of Appell sets is the existence of a (formal) power series $A(t) = \sum_0^\infty \alpha_m t^m$ $\alpha_0 \neq 0$ such that

$$(1.1) \qquad \sum_0^\infty p_n(x)t^n = A(t)e^{xt}.$$

Sheffer [2] and Steffensen [3] generalized Appell sets by considering a linear differential operator of infinite order with constant coefficients

$$(1.2) \qquad L(D) = \sum_{k=0}^\infty C_k\, D^{k+1}, \qquad C_0 \neq 0,$$

as a generalization of the differential operator D. Both consider p.s. which satisfy

$$(1.3) \qquad L(D)p_n(x) = p_{n-1}(x).$$

Sheffer proved that, among other characterizations, polynomial sets have the property (1.3) if and only if they have a generating function of the form

$$(1.4) \qquad A(t)e^{xH(t)} = \sum p_n(x)t^n$$

where

$$H(t) = \sum_{i=0}^\infty h_i t^{i+1}, \qquad A(t) = \sum_{i=0}^\infty \alpha_i t^i$$

$\alpha_0 h_0 \neq 0$.

Recently Osegov [2] has generalized Appell sets in a different direction. He studies polynomial sets which have the property

$$(1.5) \qquad D^r p_n(x) = p_{n-r}(x) \qquad (n = r, r+1, \cdots)$$

where r is a (fixed) positive integer.

In this note we consider a class of polynomials which contain both the Sheffer–Steffensen sets and the Osegov sets of polynomials. We shall obtain characterizations analogous to (1.1), (1.4) and to theorems obtained by Thorne [5] Osegov [2] and Sheffer [3].

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