TOTAL CENTRAL CURVATURE OF CURVES

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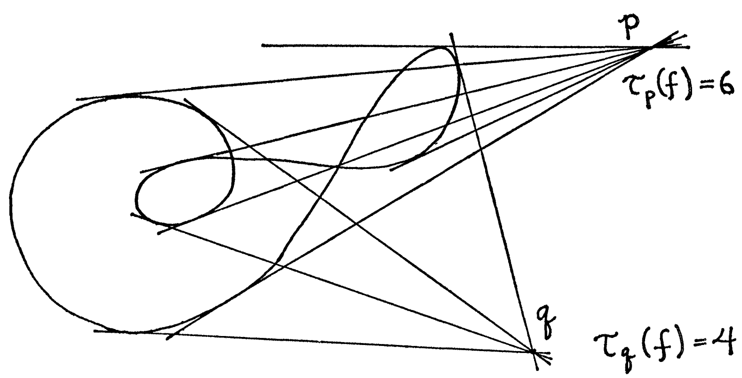
Total central curvature refers to the measure of curvedness of a space curve contained in a ball (bounded by a sphere) obtained by averaging the total absolute curvatures of the image curves under central projection from all points on the sphere. The major object of this paper is to show that this total curvature coincides with the classical total absolute curvature of the original space curve. This result generalizes immediately to curves in n-space. As a corollary we show that a curve on S^3 in E^4 with total absolute curvature < 4 in E^4 can be unknotted in S^3.

We begin by studying, from an elementary standpoint, the specialization of this theorem to plane curves, and illustrate at the same time the methods to be used in the general case.

1. Total central curvature of plane curves. Let $f\colon S^1 \to E^2$ be a continuous map of the circle S^1 into the plane. A local support line to f at x is a line containing x and bounding a closed half-plane which contains the image of a neighborhood of x in S^1. Let $\tau_p(f)$ be the number of local support lines to f passing through the point p of E^2.



The *curvature of f with respect to a circle $C = \tau_C(f)$* is defined to be the average value of $\tau_p(f)$ for points p on C, i.e.

$$\tau_C(f) = \frac{1}{l(C)} \int_{p \varepsilon C} \tau_p(f)\, ds_C$$

where ds_C denotes the element of arc of C so that $\int ds_C = l(C) =$ circumference of the circle C.

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