CONCERNING UNIVERSAL FIBRATIONS AND A THEOREM OF E. FADELL

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I. Introduction. In [9] E. Fadell showed that, assuming the base space "nice" enough, any Hurewicz fibration is fiber homotopy equivalent to a Steenrod fiber bundle, the group and fiber of the bundle depending, *a priori*, on the given fibration. One of the motivations for this note was the thought that Fadell's result might be true for arbitrary base spaces perhaps at the cost of some restrictions on the fibers. Clearly there is no need to consider fibrations without local cross-sections, so in this contex the most natural notion of fibration is one proposed by Fadell in [8], i.e., a triple $\mathcal{E} = (E, p, B)$ so that over each member of a covering $\{V_\alpha\}$ of B, E is fiber homotopy equivalent to a product $V_\alpha \times F$. In order to apply the techniques of [5] we require the covering $\{V_\alpha\}$ to be numerable. Over a CW complex this notion coincides with that of a Hurewicz fibration up to fiber homotopy type. In this setting Fadell's theorem is seen to hold (and in a more functorial way) for arbitrary fibrations with fiber dominated by a finite CW complex.

Now, our notion of fibration is in a way the analog of Dold's numerable fiber bundles [5; 248], and this leads to our second motivation namely, does Dold's classification theorem for numerable (principal) bundles [5, Theorem 7.5] have an analog for our type of fibrations? Our notion of equivalence being fiber homotopy equivalence. Part of the problem is that it is not clear what the analog ought to be, i.e., what space are we to associate to a fibration $\mathcal{E}$ so that its contractibility will imply the universality of $\mathcal{E}$? Our approach, that of using the total space of the associated principal fibration is not very satisfactory (see Theorem 3.3 and the remarks following Theorem 3.1) since it requires a strong restriction on the fibers (Theorem 4.1).

Finally it should be pointed out that we leave untouched the question of existence of universal fibrations in general simply making use of previous results ([6], see also [1] or [13]). Also, in terms of universal fibrations our definition of fibration is justified by Theorem 3.2.

II. Preliminaries and definitions. Recall from [5] that a cover $\{\mathbf{V}_\alpha\}$, $\alpha \in \mathcal{A}$ of a topological space is called numerable if it admits a refinement by a locally finite partition of unity. It will be convenient to have symbols for the following categories

$\mathcal{C}_0$ = category of topological spaces and homotopy classes of continuous maps.

$\mathcal{C}_1$ = full subcategory of $\mathcal{C}_0$ whose objects are spaces X admitting a numerable cover $\{V_\alpha\}$ such that the inclusions $V_\alpha \subset X$ are null homotopic.

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