THE RELATION OF BREADTH AND CO-DIMENSION IN TOPOLOGICAL SEMILATTICES

By Jimmie D. Lawson

In [1] L. W. Anderson has shown that in a locally compact, chain-wise connected, distributive topological lattice L the breadth of L is bounded by the codimension of L. The primary purpose of this paper is to derive a similar result for locally compact, chain-wise connected topological semilattices. It is a pleasure to acknowledge the advice and encouragement of Professor D. R. Brown.

1. Preliminaries. If X is a topological space and A is a subset of X, then A^* denotes the closure of A and A^0 denotes the interior of A. If $x \in X$, A is a *neighborhood* of x if $x \in A^0$.

A *partially ordered topological space* is a pair $(X, \leq)$ such that X is a topological space and $\leq$ is a partial order on X whose graph is closed in $X \times X$. If A is a subset of X, then

$$L(A) = \{x : x \leq a \text{ for some } a \in A\}$$
$$M(A) = \{y : a \leq y \text{ for some } a \in A\}$$
$$C(A) = L(A) \cap M(A).$$

If $x, y \in X$, then $[x, y] = M(x) \cap L(y)$. A subset B of X is *convex* (*order-dense*) if $x, y \in B$ implies $[x, y] \subset B([x, y] \cap B$ does not have cardinality 2). If the open, convex subsets of X form a basis for the topology, X is *locally convex.* The smallest convex subset of X containing a subset A is $C(A)$.

A *topological semilattice* (henceforth denoted TSL) is a pair $(S, \leq)$ such that S is a Hausdorff topological space, $\leq$ is a semilattice ordering on S, and the function from $S \times S$ into S carrying (x, y) into inf $\{x, y\}$ is continuous. We denote inf $\{x, y\}$ by $x \wedge y$. Equivalently, S is a commutative, idempotent topological semigroup with $x \leq y$ if and only if $xy = x$.

A *chain* is a totally ordered subset of S. If for each $x, y \in S$ such that $x < y$ there is a compact, connected chain with endpoints x and y, then S is *chain-wise connected.* Compact, connected, metric chains are known to be arcs [13] and will henceforth be called *arc chains.* The letter I will denote the topological semilattice defined on the arc $[0, 1]$ by $x \wedge y = \min \{x, y\}$. Any arc chain is topologically isomorphic to I.

If S is a semilattice, the *breadth of* S, denoted $Br(S)$, is the smallest integer n

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