ACTIONS WITH TOPOLOGICALLY RESTRICTED STATE SPACES

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If T is a topological semigroup and X is a Hausdorff space, an *act* is a continuous function $\mu: T \times X \to X$ such that $\mu[s, \mu(t, x)] = \mu(st, x)$ for each $s, t \in T$ and $x \in X$. We say that T *acts* on X and refer to X as the *state-space* of the act. If $S \subset T$ and $A \subset X$, we write SA rather than $\mu(S, A)$ when no confusion is likely. For $t \in T$, statements such as t is a constant map or t is a homeomorphism on $A \subset X$ refer to properties of the associated map $x \to tx$. We say that T acts *unitarily* on X iff $x \in Tx$ for each $x \in X$. An exposition of the elementary properties of acts is available as §1 of [7].

Our topological semigroup notation will be standard, usually following that found in [8]. A continuum semigroup with identity will be called a clan. We shall use $E(T)$ to denote the set of idempotents in the semigroup T and $K(T)$ to denote the minimal ideal (if it exists). $\mathcal{H}$ denotes the equivalence relation on T defined by Green as follows:

$$t\mathcal{H}s \text{ iff } t \cup tT = s \cup sT \quad \text{and} \quad t \cup Tt = s \cup Ts.$$

The $\mathcal{H}$-class of $t \in T$ will be denoted by $H_T(t)$ or, when no confusion is likely, simply by $H(t)$.

We consider only Hausdorff topological spaces. By $X = B \mid C$ we mean that X is the union of the non-void separated sets B and C. The boundary of A is denoted by $F(A)$, the interior by A^0, the closure by A^*, and $\emptyset$ is the void set. $\mathcal{J}$ denotes the class of continua irreducible between two points.

A study was made, about ten years ago, of continua in $\mathcal{J}$ which support a continuous, associative, onto multiplication [5]. Here we consider the broader question of what can be said about an act whose state-space belongs to $\mathcal{J}$. Day and Wallace [3] have completed such an inquiry under the assumption that the state-space X contains an open dense half-line with non-degenerate complement and that the action is unitary and satisfies $\emptyset \neq \{x \mid Tx = X\} \neq X$. Remark 1 (at the end of this paper) shows that their Theorem 1.8 follows directly from our

THEOREM. *Let the continuum semigroup T act unitarily on the continuum X irreducible between two of its points. Then $X/K(T)X$ is an arc or a point. If B is a component of $X - K(T)X$, B^* admits the structure of an abelian semigroup with identity and $F(B) = K(B^*)$.*

Moreover, $K(T)X$ iz irreducible between two of its points or it supports the structure of an abelian topological group. The latter must be the case if $K(T)X$ has vacuous interior.

Finally, $K(T)$ consists entirely of constant maps or entirely of homeomorphisms on $K(T)X$.

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