# LOCALLY ALGEBRAICALLY INDEPENDENT COLLECTIONS OF SUBSEMIGROUPS OF A SEMIGROUP

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**1. Introduction.** Given topological semigroups $T_1, T_2, \cdots, T_n$ with identity elements $u_1, u_2, \cdots, u_n$, respectively, the cartesian product semigroup $T_1 \times T_2 \times \cdots \times T_n$ (with coordinate-wise multiplication) may be realized as the pointwise product of the subsemigroups $\{u_1\} \times \cdots \times T_i \times \cdots \times \{u_n\}$, $(i = 1, 2, \cdots, n)$, each having identity element $(u_1, u_2, \cdots, u_n)$. In this paper a partial converse of this statement is considered: given a topological semigroup $S$ with identity element 1 and subsemigroups $T_1, \cdots, T_n$ each having identity element 1, under what conditions is the pointwise product $T_1 \cdots T_n$ isomorphic (or at least locally isomorphic in a neighborhood of 1) to the cartesian product semigroup $T_1 \times \cdots \times T_n$? It is immediately apparent that some condition of "independence" must be imposed on the collection $\{T_1, \cdots, T_n\}$ of subsemigroups. In this paper attention is restricted to the local situation and the concept of a locally algebraically independent collection of subsemigroups is introduced. For particular classes of compact topological semigroups, this condition imposed on the collection $\{T_1, \cdots, T_n\}$ in the situation described above is sufficient to guarantee the existence of a homomorphism from $T_1 \times \cdots \times T_n$ onto $T_1 \cdots T_n$ which is one-to-one in some neighborhood of the identity element in $T_1 \times \cdots \times T_n$. Thus the pointwise product is locally isomorphic to the cartesian product. The main theorems in this connection are Theorems 3.1 and 3.3.

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**2. Preliminaries.** By the term "semigroup" we will mean topological semigroup. By a homomorphism of a semigroup $S$ into a semigroup $T$ we will mean a continuous function $f$ from $S$ into $T$ which is an algebraic homomorphism. If $f$ is one-to-one and has a continuous inverse, $f$ will be called an isomorphism.

The $\mathcal{H}$-relation in a semigroup $S$ with an identity element is defined by

$$\mathcal{H} = \{(x, y) \in S \times S \mid xS = yS \text{ and } Sx = Sy\}.$$

If $S$ is compact and commutative, then $\mathcal{H}$ is a closed congruence and the quotient semigroup $S/\mathcal{H}$ is a compact topological semigroup. The $\mathcal{H}$-class containing an element $x$ will be denoted by $H(x)$. The $\mathcal{H}$-class of an idempotent is a group and, if $S$ is compact, a compact topological group. If $S$ has an identity element 1, then $H(1)$ acts as a transformation group on $S$ by $(h, x) \to hx$. We will say that the action of $H(1)$ on $S$ is *locally effective* if there exists a neighborhood $V$ of $H(1)$

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