ON A QUESTION OF WOJDYSLAWSKI AND STROTHER

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1. Introduction. The following question was essentially raised by Wojdyslawski and stated by Strother [7]: If X is connected and compact Hausdorff, is 2^X an absolute extensor for the class of normal spaces, where 2^X is the space of nonempty closed subsets of X with the finite topology (The family of all subsets of 2^X of the form

$$\langle V_1, \cdots, V_n \rangle = \left\{ A \in 2^X \mid A \subset \bigcup_{i=1}^{n} V_i \quad \text{and} \quad A \cap V_i \neq \varnothing \quad \text{for} \quad \text{each } i \right\}$$

is a base for the finite topology)? Strother [8; Theorem 8 and Lemma d] answered this question negatively by proving the following result: If X is a connected compact metrizable space, then 2^X is an absolute extensor for the class of normal Hausdorff spaces if and only if X is locally connected. Strother's result immediately suggests a refinement of Wojdyslawski's question: *Is a compact Hausdorff, connected and locally connected space an absolute extensor space for the class of normal (or compact) Hausdorff spaces?* Unfortunately, the answer to this question is "no", even if one requires that all homotopy groups $\pi_n(X)$ of X be trivial for $n = 1, 2, \cdots$.

In the process of answering the preceding question we seem to develop a fairly general technique of getting counterexamples to various appealing conjectures concerning multivalued functions. For example, we will also show that if $F, G : X \to Y$ are multivalued functions, F is lsc (i.e. $F^{-1}(V)$ is open for each open $V \subset Y$), G is usc (i.e. $F^{-1}(B)$ is closed for each closed $B \subset Y$) and $F \subset G$ (i.e. $F(x) \subset G(x)$ for each $x \in X$) then not always can one find a continuous function $H : X \to Y$ (i.e. H is lsc and usc) such that

$$F \subset H \subset G,$$

even if X is metrizable and Y is compact, connected and metrizable. If such a function H could always be found then, due to some results of [2], we would get a very powerful result on the extension of continuous multivalued functions (see Theorem 4.4).

2. Preliminary results. Throughout we will use the terminology of [2], except that we will only consider *point-closed* multivalued functions $F : X \to Y$ (i.e. $F(p)$ is a closed nonempty subset of Y for each $p \in X$). Clearly, all single-valued functions are point-closed. For the sake of unity, we make the following definition:

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