SOME GENERATING FUNCTIONS FOR LAGUERRE POLYNOMIALS

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Put

$$L_n^{(a)}(x) = \sum_{k=0}^{n} (-1)^k \frac{(a+1)_n x^k}{(a+1)_k k!\,(n-k)!}. \tag{1}$$

Employing a formula due to Chatterjea [2], J. W. Brown [1] has proved that $L_n^{(a+mn)}(x)$ satisfies a generating relation of the form

$$\sum_{n=0}^{\infty} L_n^{(a+mn)}(x)t^n = A(t) \exp(xB(t)), \tag{2}$$

where m is an arbitrary integer. Moreover given (2) for m negative, it is shown that

$$\sum_{n=0}^{\infty} L_n^{(-a-(m+1)n)}(x)t^n = \frac{A(-t)}{1-B(-t)} \exp\left\{\frac{-xB(-t)}{1-B(-t)}\right\}. \tag{3}$$

It may be of interest to point out that a result of the type (2) holds when m is an arbitrary complex number; similarly for (3).

Consider the sum

$$\begin{aligned}\sum_{n=0}^{\infty} L_n^{(a+bn)}(0)t^n &= \sum_{n=0}^{\infty} \frac{(a+bn+1)_n}{n!} t^n \\ &= \sum_{n=0}^{\infty} \binom{a+(b+1)n}{n} t^n.\end{aligned}$$

By the Lagrange expansion formula [3, vol. I, p. 126, ex. 212] we have

$$(1+v)^{a+1} = 1 + (a+1) \sum_{n=1}^{\infty} \binom{a+(b+1)n}{n-1} \frac{t^n}{n}, \tag{4}$$

where

$$v = t(1+v)^{b+1}, \qquad v(0) = 0. \tag{5}$$

Differentiating (5) we get

$$\frac{dv}{dt} = \frac{(1+v)^{b+2}}{1-bv}; \tag{6}$$

now differentiating (4) and making use of (6) we find that

$$\frac{(1+v)^{a+1}}{1-bv} = \sum_{n=0}^{\infty} \binom{a+(b+1)n}{n} t^n. \tag{7}$$

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