STOPPING TIMES FOR RECURRENT STABLE PROCESSES

By Sidney C. Port and Charles J. Stone

1. Introduction. Throughout this paper $X(t)$ will denote a recurrent stable process on R^d, having exponent α, transition density $p(t, x)$, and paths which are normalized to be right continuous with left-hand limits at every point. In the sequel we shall, without further ado, use consequences of the well-known fact that all such processes are strong Markov processes. Our main purpose in this paper is to investigate the asymptotic behavior, for large t, of the quantity $P_x(T > t)$ where T is a stopping time for the $X(t)$ process satisfying Conditions 1 and 2, stated below. Previously, this quantity was investigated in [2] for the hitting times T_B of compact sets B such that $P_x(T_B < \infty) \equiv 1$. The methods used here are similar to those used in [2], and the results we obtain here are analogous to those for the hitting times T_B. The conditions we shall impose on the stopping times T are as follows:

Condition 1. (*Compact stopping place*). *There is a compact set* B *such that*

$$P_x(T < \infty, X(T) \notin B) \equiv 0.$$

Condition 2. (*Uniform lower bound*). *For each compact set* C *there is a* t_0 *and* $\delta > 0$ *such that for all stopping times* σ

$$P(T < t_0 + \sigma \mid \mathfrak{F}_\sigma) \geq \delta, \quad \text{a.e. on} \quad [\sigma < \infty, X(\sigma) \in C],$$

where $\mathfrak{F}_\sigma$ *denotes the usual* σ*-field associated with the stopping time* σ.

Although at first glance Condition 2 looks a bit forbidding, we will show below that it is satisfied for a large variety of stopping times T. In particular, Example 1 will show that if T_B is the first hitting time of a bounded Borel set B, then T_B satisfies our conditions, and thus the results we obtain here will include those of [2]. Henceforth T will be an arbitrary stopping time satisfying Conditions 1 and 2 above.

We note that it follows from our conditions (see §2) that $P_x(T < \infty) \equiv 1$. For $\lambda > 0$ set

$$G_T^\lambda(x, dy) = \int_0^\infty e^{-\lambda t} P_x(X(t) \in dy, T > t)\, dt.$$

For $\alpha = d$ (i.e. $\alpha = 1$, $d = 1$ or $\alpha = 2$, $d = 2$) and B as in Condition 1, set

$$D_x = \{y \in R^d : y \notin B \cup \{x\}\},$$

while for $\alpha > d$ (i.e. $1 < \alpha \leq 2$, $d = 1$) set $D_x = R^1$. It is not difficult to show

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