LATTICES OF SEQUENCE SPACES

By William Ruckle

1. Introduction. Let E be a Hausdorff linear topological space and X and Y linear subspaces of E upon which are defined locally convex topologies stronger than the topology induced by E. There is a natural way (Definition 2.5) to define locally convex topologies on the linear spaces $X + Y$ and $X \cap Y$. This paper discusses properties which are preserved by sum and intersection, in particular when E is the space ω of all scalar (real or complex) sequences with the topology of coordinatewise convergence.

The sequence whose ith term is x_i is denoted by (x_i) or simply x. The sequence with 1 in the n-th place and 0's elsewhere is written e_n; the set $\{e_1, e_2, \cdots\}$ is written $\mathcal{E}$. The sequence spaces which are considered here are all assumed to contain the set $\mathcal{E}$ and hence the set φ of all sequences which are zero except in a finite number of coordinates.

1.1 Definition. A sequence space X which contains φ is a K-space if it is a locally convex Hausdorff space on which the coordinate functionals defined by $E_i(x) = x_i$ are continuous. In other words the locally convex topology on X is stronger than the relative topology of X as a subspace of ω.

The set $E_1, E_2, \cdots$ is denoted by $\mathcal{E}'$.

A K-space X is an FK-space if it is an F-space (complete metric) as well. It is a BK-space if it is a Banach space.

The term K-space is used in [5] where such spaces are studied in detail; FK-spaces are treated in [12].

Six BK-spaces which are mentioned in the course of this paper are:

c_0, the space of sequences which converge to 0;

l, the space of absolutely convergent series;

bv, the space of all sequences x for which $||x|| = \sum_{i=1}^{\infty} |x_{i+1} - x_i| + |\lim_n x_n|$ is finite;

bv_0, the closed linear span of $\mathcal{E}$ in bv;

bs, the space of all sequences x for which $||x|| = \sup_n |\sum_{i=1}^{n} x_i|$ is finite;

cs, the closed linear span of $\mathcal{E}$ in bs.

These spaces are discussed in IV.2 of [4].

2. Preliminary results on the sum and intersection of subspaces. If p is any seminorm defined on a subspace X of a linear space E, its domain can be ex-

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