INEQUALITIES FOR SYMMETRIC FUNCTIONS

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1. In [1] it is proved that if E_r is the r-th elementary symmetric function in the n-variables $\alpha_1, \alpha_2, \cdots, \alpha_n$, then

$$E_r E_s \geq E_{r-1} E_{S+1} \qquad (S \geq r) \tag{1.1}$$

where $\alpha_1, \alpha_2, \cdots, \alpha_n$, are non-negative reals.

In [2], defining

$$p_r = \frac{E_r}{\binom{n}{r}} \tag{1.2}$$

a similar relation is proved for the p_r-function.

In [5], Doughall has given additional properties of the same function. We define

$$S_r = \alpha_1^r + \alpha_2^\gamma + \cdots + \alpha_n^r \tag{1.3}$$

$$H_r = \sum \alpha_1^{c_1'} \alpha_2^{c_2'} \cdots \alpha_n^{c_n'} \tag{1.4}$$

$$c_1' + c_2' + \cdots + c_n' = r$$

$$0 \leq c_1' \leq c_2' \leq \cdots \leq r$$

$$h_r = \frac{H_r}{\binom{r+n-1}{n-1}}; \tag{1.5}$$

in other words, h_r is the function obtained by dividing H_r by the number of terms in H_r.

In this paper we discuss similar relations for H_γ, h_γ, S_r. We prove that

$$H_{p-\lambda} H_{q+\lambda} \geq H_{p-\lambda-1} H_{q+\lambda+1} \qquad (q \geq p) \quad (0 \leq \lambda < p) \tag{1.6}$$

also

$$S_{r-\mu} S_{t+\mu} \leq S_{r-\mu-1} S_{t+\mu+1} \qquad (t \geq \gamma) \quad (0 \leq \mu < \gamma). \tag{1.7}$$

Other properties of these functions are also discussed. We prove that

$$h_{p-\lambda} h_{q+\lambda} \geq h_{p-\lambda-1} h_{q+\lambda+1} \qquad (q \geq p) \quad (0 \leq \lambda < p) \tag{1.8}$$

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