SOME NOTES ON SEQUENCES WHICH ARE SIMILAR OR RELATED TO A SCHAUDER BASIS

By Lee W. Baric

In this paper we propose to study certain properties of sequences in a Banach space which are similar to a Schauder basis for the space. We find that, under suitable conditions concerning the existence of functionals biorthogonal to such a sequence, the sequence is basic if and only if a particular sequence of linear operators is pointwise bounded on the linear closure of the sequence. Related sequences are defined, and a result is stated concerning the conditions under which the transpose of the Cesàro matrix of order one transforms a sequence related to Schauder basis into a basic sequence.

Throughout this paper X will denote a Banach space, $\mathfrak{X} = \{x_i\}$ a Schauder basis for X and $\mathfrak{F} = \{f_i\}$ the sequence of continuous linear functionals biorthogonal to $\mathfrak{X}$. Hereafter, Schauder basis will be abbreviated to basis.

1. A result on sequences similar to a basis. Recall that two sequences $\mathcal{Y} = \{y_i\}$ and $\mathfrak{z} = \{z_i\}$ in X are called similar if and only if $\sum_i t_i y_i$ and $\sum_i t_i z_i$ are either both convergent or both divergent for any given scalar sequence $t = \{t_i\}$, [1].

Suppose $\mathcal{Y}$ is a sequence in X which possesses continuous biorthogonal linear functionals $\mathcal{G} = \{g_i\}$. For any positive integer n, let

$$T_n(x) = \sum_{i=1}^{n} f_i(x) y_i ,$$

and

$$S_n(x) = \sum_{i=1}^{n} g_i(x) x_i \quad \text{for} \quad x \in X.$$

Also let

$$D_T = \{x : x \in X \text{ and } \lim_n T_n(x) \text{ exists}\}$$

and

$$D_S = \{x : x \in X \text{ and } \lim_n S_n(x) \text{ exists}\}.$$

For $x \in D_T$, let $T(x) = \lim_n T_n(x)$, and for $x \in D_S$ let $S(x) = \lim_n S_n(x)$. Throughout the remainder of this section we shall assume that $\mathcal{Y}$ and $\mathcal{G}$ satisfy these conditions.

Received November 7, 1966. This work was in part supported by a grant from the Faculty Research Fund of Dickinson College.