THE COMMUTATOR GROUP OF A DOUBLED KNOT

By C. B. Schaufele

1. Introduction. If the commutator group of a tame knot is not finitely generated, then it is an infinite free product with amalgamation on a free group of rank twice the genus of the knot [3] and [1]. In [4], Neuwirth asks if this decomposition is unique in the sense that the amalgamating subgroup always has rank twice the genus of the knot. The purpose of this paper is to provide a negative answer to this question. It will be shown that if k is a non-trivial tame knot with finitely generated commutator group, then the double of k [5] has commutator group which can be expressed as an infinite free product with amalgamation in two ways, one where the amalgamating group is free of rank 2, and the other where the amalgamating group is free of rank $4g$, where g is the genus of k.

If k is a knot with group G, then a covering space for $S^3 - k$ corresponding to the commutator group $[G, G]$ of k can be constructed by choosing any orientable surface $\mathring{S}$ which spans k and gluing together copies of $S^3 - (k \cup$ regular neighborhood of $\mathring{S})$ in the manner specified in [3]. If S is such that the inclusion $\mathring{S} \subset S^3 - k$ induces a monomorphism $\pi_1(\mathring{S}) \to G$, then Van Kampen's theorem can be used on the covering space to determine the structure of $[G, G]$. Henceforth, we will call such a spanning surface for k *injective*. Every knot k has an injective spanning surface, namely, an orientable surface of minimal genus which spans k. The only injective surface for a knot with finitely generated commutator group is the surface of minimal genus which spans the knot, for the proof of Neuwirth's structure theorem shows in this case that if S is injective, $[G, G] \approx \pi_1(S)$. On the other hand, if $[G, G]$ is not finitely generated and S is injective, we can use S to construct the covering space of $S^3 - k$ corresponding to $[G, G]$ and apply Van Kampen's theorem to obtain the decomposition $[G, G] = \cdots \underset{F}{*} H \underset{F}{*} H \underset{F}{*} \cdots$, where $H \approx \pi_1(S^3 - S)$ and $F \approx \pi_1(S) =$ free group of rank twice the genus of S. Thus, in order to provide a negative answer to Neuwirth's question, we must find a knot k which has an injective spanning surface of genus greater than the genus of k.

Throughout this paper, k will denote a nontrivial knot of genus g, tamely embedded in S^3, for which $G = \pi_1(S^3 - k)$ has finitely generated commutator group $[G, G]$ (and hence free of rank $2g$); dk will denote the double of k with group $G_d = \pi_1(S^3 - dk)$; S will be an orientable surface of genus g, which spans k; and F_r will denote the free group of rank r.

Everything will be assumed to be simplicial.

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