A CONSEQUENCE OF THE NON-EXISTENCE OF CERTAIN GENERALIZED POLYGONS

BY STANLEY E. PAYNE

Introduction. Let M be a collection of v points and v lines with S lines through each point, S points on each line, $S \geq 3$. An *n-gon* of M is a collection of n distinct points $x_1, \cdots, x_n$ and n distinct lines $L_1, \cdots, L_n$ of M such that $x_i \in L_i \cap L_{i+1}$ for $1 \leq i \leq n-1$, and $x_n \in L_n \cap L_1$. If K is the smallest positive integer n such that there is an n-gon of M, then M is said to be a $v \times v(K, S)$-*configuration*, and $v \geq \sum_{i=0}^{k-1} (S-1)^i$ [2]. If equality holds, M is said to be *projective* and is in the class of generalized K-gons for which Feit and Higman have shown that $K = 3, 4$, or 6 [1]. Our main result is the following consequence of the non-existence of a projective $(5, S)$-configuration:

THEOREM 1. *Let $S-1$ be a prime power and let F and F' be the fields with $S-1$ and $(S-1)^2$ elements, respectively, $F \subset F'$. Then there do not exist elements $e_1, \cdots, e_S, e_1', \cdots, e_S'$ of F' with $e_1, \cdots, e_S$ distinct and such that all three of the following implications hold*:

A. *For $a \in F'$, if $a(e_{t_1}' - e_{t_1}) \in F$ and $a(e_{t_2} - e_{t_1}) \in F$, then $a = 0$ or $t_1 = t_2$.*

B. *For $a, c \in F'$, if $ae_{t_1} - ce_{t_2} = (a-c)e_{t_3}$, $c(e_{t_2}' - e_{t_2}) \in F$, and $(a-c)(e_{t_3}' - e_{t_3}) \in F$, then either $a = c = 0$ or $t_1 = t_2 = t_3$.*

C. *For $a_i \in F'$, $i = 1, 2, 3, 4$, if $\sum_{i=1}^{4} a_i = 0$ and $a_i(e_{t_i}' - e_{t_i}) \in F$, then either $a_i = 0$ for all i or $t_1 = t_2 = t_3 = t_4$.*

For $S \geq 4$ there is the

COROLLARY. *Let ξ be a generator of the multiplicative group of F'. Then for $i = 1, 2, 3, 4$ there exist $a_i \in F'$ not all zero and distinct integers t_i, $1 \leq t_i \leq S$ such that $\sum a_i = \sum a_i \xi^{t_i} = 0$, and such that $a_i \xi^{t_i} \in F$ for each $i = 1, 2, 3, 4$.*

Preliminaries. Lines or points of M will be called *elements* of M. For elements T, T' of M put $d(T, T') = 0$ if $T = T'$, $d(T, T') = 1$ if one of T, T' is on the other. If n is the smallest integer r such that there are elements $T_1, T_2, \cdots, T_r = T'$ with $d(T, T_1) = d(T_1, T_2) = \cdots = d(T_{r-1}, T_r) = 1$, put $d(T, T') = n$. Then for projective M: we list the following relations for reference:

1) $d(T, T') \leq K$ for any elements T, T' of M.

2) If $d(T, T') < K$ there is a unique sequence of elements determining $d(T, T')$.

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