THE EILENBERG-BORSUK DUALITY THEOREM FOR METRIC SPACES

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1. In [3], S. Eilenberg proved the following useful 'théorème de dualité' on extensions of maps (that is, continuous functions).

Theorem. *Suppose A is a closed set of a compact metric (hence separable) space X with* $\dim (X \backslash A) \leq n$. *Then for each map $f: A \to S^k$ from A to the k-dimensional sphere, there exists a set $E \subset X \backslash A$, E closed in X with* $\dim (E) < n - k$ *such that f can be extended over $X \backslash E$.* ($X \backslash A$ denotes the set-theoretic difference.)

K. Borsuk showed [1] that the theorem of Eilenberg remained valid under the weaker assumption that the domain be a separable metric space and the range be a separable metric space which is $(k - 1)$-connected and locally $(n - 1)$-connected.

By an *absolute neighborhood retract* (ANR) we mean a space Y in the class M of all metrizable spaces such that every homeomorphic image of Y as a closed subspace of a space Z in M is necessarily a neighborhood retract of Z. Recent investigations have prompted the choice of our preferred class M for the definition of ANR [6]. On the one hand, ANR's for the class M can be ANR's for larger classes of spaces if suitable condition of completeness, separability, or compactness is satisfied. On the other hand, ANR's for the class of compact metrizable spaces and the class of separable metrizable spaces are merely special cases of ANR's for the class M. No other weakly hereditary topological class of spaces has such a favorable position.

The objective of this paper is to prove the above-mentioned theorem without the separability condition if the covering definition of dimension is used. We first prove the theorem for maps from a metric space to the k-dimensional sphere S^k by arguments as in [1]. Then we are able to prove the theorem for maps from a metric space into a metric ANR by means of S. T. Hu's bridge theory technique [5]. This method of proof enables us to generalize for metric ANR's some classical theorems which were proved for separable metric ANR's by Hu [5].

2. A metric space X is said to be of dimension $\leq n$, in symbol, $\dim X \leq n$ if and only if every open covering α of X has a locally finite open refinement β such that its nerve K_β contains no simplex of dimension greater than n.

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