NOTE ON AUTOMORPHIC FORMS WITH REAL PERIOD POLYNOMIALS

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1. Introduction. Recently a certain amount of interest has been engendered in looking upon autormorphic forms with so-called "period polynomials" as a generalization of abelian integrals with periods (see [1], [3], [4], [5]).

Such forms can arise as follows: Let Γ be an H-group (horocyclic group as defined in [7; 265] or [5; 168]) and let $G(\tau)$ be a regular automorphic form in τ belonging to Γ and having dimension $-r - 2$ (r a positive even integer). That is, $G(\tau)$ is analytic in the upper half-plane without singularities at the cusp points and

$$(1) \qquad \begin{cases} (c\tau + d)^{-r-2}G(V\tau) = G(\tau), & \text{Im} \quad \tau > 0 \\ V = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \varepsilon\ \Gamma, & ad - bc = 1. \end{cases}$$

(Throughout we will always take a, b, c, d real, as is always possible with an H-group.) Now the forms with "period polynomials" are obtained by taking a "general" $(r + 1)$-th primitive of $G(\tau)$, say "integrating from τ_0,"

$$(2) \qquad H(\tau) = \int_{\tau_0}^{\tau} G(\zeta)(\tau - \zeta)^r\, d\zeta/r! + h_0(\tau) \qquad (\text{Im}\ \tau_0 > 0)$$

where $h_0(\tau)$ is a polynomial of degree $\leq r$. It can then be verified by actual substitution that, in contrast to (1),

$$(3) \qquad \begin{cases} (c\tau + d)^r H(V\tau) = H(\tau) + h_V(\tau), & \text{Im} \quad \tau > 0 \\ V = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \varepsilon\ \Gamma \end{cases}$$

where $h_V(\tau)$ is the following polynomial (of degree $\leq r$ again):

$$(4) \qquad h_V(\tau) = \int_{V^{-1}\tau_0}^{\tau_0} G(\zeta)(\tau - \zeta)^r\, d\zeta/r! + (c\tau + d)^r h_0(V\tau) - h_0(\tau),$$

with $h_0(\tau)$ the arbitrary polynomial in (2). A function $H(\tau)$ satisfying (3) will be said to be an automorphic form of (positive) dimension r with *period polynomials* $h_V(\tau)$. The repeated application of (3) yields the consistency relation

$$(5) \qquad h_{V_1V_2}(\tau) = h_{V_1}(V_2\tau)(c_2\tau + d_2)^r + h_{V_2}(\tau),$$

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