SEMIGROUPS, CONTINUA AND THE SET FUNCTIONS T^n.

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1. **Introduction.** In [1] we generalized Jones' property in [5] of aposyndetic and obtained, for $A \subset S$, a set $T(A)$ defined as follows: $S - T(A) = \{x \mid$ there exist a continuum W and an open set Q such that $S - A \supset W \supset Q \supset x \in S\}$. We say that $T^0(A) = A$ and $T(T^{n-1}(A)) = T^n(A)$, which defines the set functions T^n $(n = 0, 1, 2, \cdots)$. In [1] we obtained various properties of T^n. Here we continue the study of these for both semigroups and continua.

The concept of $T(p)$ is used by Koch and Wallace in [7; 280] and is basic in Hunter's work in [4]. Another concept found to be useful in the study of semigroups is that of C-set. See for example Wallace in [11]. We prove several theorems concerning its relation to T^n.

Koch and Wallace in [7] show that if $S = SS$ and if S is a semigroup and continuum irreducible from the minimal ideal K to A, then the Rees Quotient S/K is irreducibly connected between zero and unit, that is, it is an arc. They use a theorem by Mostert and Shields to do this. Below we give a more set-theoretic proof of this, without using Mostert and Shields' results; but Faucett's Theorem 2.2 in [2] is basic in this proof; in this we need to assume that, if p is a limit point of a set M, then p is a sequential limit of a subsequence of M. We base this proof upon the theorems given below; instead it could be based upon Hunter's theorems in [4].

We define total (n, T)-sets and show these form a class of equivalence sets whose union is S.

Our basic set-theoretic definitions are in [9] and [13]. By a continuum we mean a closed and connected set. The basic semigroup definitions are given by Wallace in [12]; by semigroup we mean a topological semigroup: that is, S is a Hausdorff space together with a continuous associative multiplication m: $S \times S \to S$. Our references are to [9], but the stronger space of [9] is not needed for our theorems. We denote the null set by ϕ and the closure of M by $\bar{M}$. We denote the minimal ideal by K and the set of idempotents by E.

2. **Ideals and (n, T)-sets.** We show that an ideal is an (n, T)-set. We recall: if L is a left ideal of S, then $L \neq \phi$ and $SL \subset L$; if R is a right ideal of S, then $R \neq \phi$ and $RS \subset R$; and if I is an ideal of S, then $I \neq \phi$ and $SIS \subset I$. We write $L(x) = x \cup Sx$; $R(x) = x \cup xS$; and $J(x) = x \cup Sx \cup xS \cup SxS$.

Let S be a compact continuum: below S then has the further properties: (a) every left ideal, which contains a right ideal, is connected; thus, if L is a left ideal,

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