THE BRAID GROUPS OF E^2 AND S^2

By Edward Fadell and James Van Buskirk

I. **Introduction.** Recently, R. H. Fox introduced a new definition of the geometric braid group of the plane E^2 and in [6] a new proof that it is isomorphic to the well-known algebraic braid group of Artin [1] is given. In [4], the concept of the braid group of an arbitrary manifold is studied. There, an associated class of fiber spaces arising naturally from the situation gives information about the homotopy groups of certain configuration spaces which in turn gives information about braid groups. In this paper we pursue this point of view and give another proof that the geometric braid group of E^2 is the algebraic braid group of E^2 and also show that the geometric braid group on S^2 is the algebraic braid group of E^2 with one additional relation. The geometric key to the proof is the fact that the second homotopy group is trivial for certain configuration spaces.

Braids on S^2 have been considered by Magnus [7] and Newman [8]. (The writers are indebted to R. H. Fox for pointing out the papers of Magnus [7] and Newman [8] and for stimulating discussions.) In [7] Magnus considers a certain automorphism group of the fundamental group of the punctured sphere and shows that it is isomorphic to a factor group of the algebraic (Artin) braid group on E^2. He gives there a geometric interpretation of the additional relations in terms of considering braids whose ends are located on concentric spheres, the inner one of which is allowed to rotate. However, the Magnus group is not the braid group on S^2 since the freedom to rotate the inner sphere introduces relations which hold in the Magnus group but do not hold in the braid group on S^2. As has already been pointed out in the preceding paper, Newman in [8] makes use of the fact that the algebraic and geometric braid groups on S^2 are isomorphic, but no proof of this is in evidence.

II. **Topological preliminaries.** 1. Let M denote a manifold (locally Euclidean connected Hausdorff space) of dimension ≥ 2 and $Q_m = \{x_1, \cdots, x_m\}$ a fixed set of m distinct points of M. The *configuration space* $F_{m,n}(M)$ is defined as follows:

$$F_{m,n}(M) = \{(p_1, \cdots, p_n) : p_i \in M - Q_m, p_i \neq p_j \quad \text{for} \quad i \neq j\}.$$

When the manifold M is fixed or otherwise understood, we will designate $F_{m,n}(M)$ simply by $F_{m,n}$. We give $F_{m,n}$ the natural topology induced by M and remark that if M is a k dimensional manifold, $F_{m,n}$ is an nk dimensional manifold. The following theorem is contained in [5].

Received June 19, 1961. The research of the first author was supported in part by the National Science Foundation under Grant NSF G-11297.