TEST SPACES FOR HOMOLOGICAL DIMENSION

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1. **Introduction.** Let X be a topological space and G an Abelian group. Let us denote by $D_*(X : G)$ the homological dimension of X with respect to G defined by means of Čech homology groups of pairs of closed subsets of X with coefficients in G. If we denote by dim X the covering dimension of X, it is obvious that $D_*(X : G) \leq \dim X$ for any space X and any group G. A topological space X is called *full-dimensional with respect to* G in case $D_*(X : G) = \dim X$. Then the following problem arises naturally:

(*) *Given an Abelian group G, what is a space which is full-dimensional with respect to the group G?*

A topological space M is called a *test space with respect to* G if a compact space X is full-dimensional with respect to G if and only if $\dim(X \times M) = \dim X + \dim M$. The object of this paper is to give an answer to the problem (*) for some important groups by constructing test spaces with respect to them.

A sequence $\mathfrak{a} = (q_1, q_2, \cdots)$ of positive integers is called a *k-sequence* if q_i is a divisor of q_{i+1} for each i and $q_i > 1$ for some i (cf. [7; 385]). If $\mathfrak{a} = (q_1, q_2, \cdots)$ is a k-sequence, we have the inverse system of groups $\{Z_{q_i} : h_i^{i+1}; i = 1, 2, \cdots\}$, where Z is the additive group of all integers, Z_q is the cyclic group with order q $(=Z/qZ)$ and h_i^{i+1} is a natural homomorphism from $Z_{q_{i+1}}$ onto Z_{q_i}. By $Z(\mathfrak{a})$ we denote the limit group of $\{Z_{q_i} : h_i^{i+1}\}$. For a prime p we denote by $\mathfrak{a}_p$ the k-sequence $(p, p^2, p^3, \cdots)$. Test spaces with respect to the group $Z(\mathfrak{a}_p)$, and generally with respect to the group $Z(\mathfrak{a})$, were constructed by Boltyanskii [1], [2] and the author [7], though they were not stated clearly. Let $Q(\mathfrak{a})$ be the Cantor manifold constructed in [7, §3] for any k-sequence $\mathfrak{a}$. The following theorem is proved by the same way as [7, Theorem].

THEOREM 1. *The Cantor manifold $Q(\mathfrak{a})$ is a test space with respect to $Z(\mathfrak{a})$.*

We shall prove the following theorems.

THEOREM 2. *Let R be the additive group of all rational numbers. Then there exists a Cantor manifold M_0 which is a test space with respect to R.*

THEOREM 3. *Let Q_p be the additive group of all rational numbers reduced* mod 1 *whose denominators are powers of a prime p. Then there exists a Cantor manifold M_p which is a test space with respect to Q_p.*

As a consequence of Theorem 3, we obtain Dyer's theorem [4].

THEOREM 5. *Let X and Y be compact spaces. If* $\dim(X \times Y) = \dim X +$

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