PROPERTIES OF CERTAIN NON-CONTINUOUS TRANSFORMATIONS

By Paul E. Long

Professors John Nash [2] and O. H. Hamilton [1] have, respectively, defined the connectivity map and the peripherally continuous transformation. Professor Hamilton's paper, along with that of Professor Stallings [4] deals primarily with fixed point properties of these transformations. This paper gives some further properties of these transformations including a condition which implies their equivalence.

We shall now recall the definitions of connectivity maps and peripherally continuous transformations.

Definition 1. A *connectivity map* from a space X to a space Y is a mapping f such that the induced map g of X into $X \times Y$, defined by $g(p) = p \times f(p)$, transforms connected subsets of X onto connected subsets of $X \times Y$.

Definition 2. A mapping f of a space X into a space Y is called *peripherally continuous* if and only if for each point $p \in X$ and each pair of open sets U and V containing p and $f(p)$, respectively, there is an open set $D \subset U$ containing p such that f transforms the boundary F of D into V.

In this paper we shall consider the spaces X and Y to be Hausdorff, unless otherwise explicitly stated. The boundary of a set D will be denoted by the symbol $F(D)$.

Theorem 1. *If $f : X \to Y$ is a peripherally continuous transformation of X onto Y and N is a closed subset of Y, then each component of $f^{-1}(N)$ is closed in X.*

Proof. Suppose, on the contrary, that E is a component of $f^{-1}(N)$ which is not closed in X. Then there exists a limit point x of E that does not belong to E. Since N is closed, there exists an open set V containing $f(x)$ but no point of N.

Since E is non-degenerate, there is an open subset U of X containing x such that $(X - U) \cap E \neq \phi$. Then there exists an open subset $D \subset U$ containing x such that $f(F(D)) \subset V$, since f is peripherally continuous. But $(X - D) \cap E \neq \phi$, and since E is connected, there are points of E in D and $X - D$. Therefore $F(D)$ contains at least one point of E, and it follows that $f(F(D))$ is not a subset of V, which is a contradiction. Thus the assumption that E is not closed is false, and the conclusion of the theorem follows.

Theorem 2. *If $f : X \to Y$ is a peripherally continuous transformation of X onto Y, N a connected subset of X, and $x \in \bar{N}$, then $f(x) \in (f(N))^{-}$.*

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