STABILITY OF CERTAIN QUASI-OPEN MAPPINGS

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1. **Introduction.** Let $f: X \rightarrow Y$ be a mapping (continuous function) of a space X into a space Y with metric ρ. Let $y \varepsilon f(X)$. Then y is said to be a stable value for f with respect to mappings of X into Y provided the following is satisfied. There exists an $e(y) > 0$ such that if $g : X \rightarrow Y$ is a mapping that satisfies $\rho(f, g, X) = \text{l.u.b. } \{\rho(f(x), g(x)) \mid x \varepsilon X\} < e(y)$, then $y \varepsilon g(X)$. Using the topological index and some of results of G. T. Whyburn [7] concerning the action of a light open mapping on a 2-manifold, it is easy to show that if $f : X \rightarrow P$ is a light-strongly open mapping defined on a region (open connected) X in a complex plane into a complex plane P, then every point in $f(X)$ is a stable value for such a mapping. In this paper similar results are obtained for certain other mappings. In particular proofs are furnished for the following.

1.1. Theorem. *Let $m : X \rightarrow P$ be a compact monotone mapping from a region X in a complex plane into a complex plane P. Suppose the interior (relative to P) of $m(X)$ is non-empty. Then each point in $m(X)$ is a stable value of m and $m(X)$ is open in P.*

1.2. Theorem. *Let $f : X \rightarrow P$ be a compact (strongly) quasi-open mapping of a simply connected region in a complex plane into a complex plane P. Then every point in $f(X)$ is a stable value of f.*

In reading the above, it is to be recalled that m is monotone provided that for any point $x \varepsilon f(X)$, $m^{-1}(x)$ is a continuum (compact, connected). A mapping $f : X \rightarrow P$ is compact if for each compact set $K \subset f(X)$, $f^{-1}(K)$ is compact. f is quasi-open provided that for any $y \varepsilon f(X)$ and any open set U in X containing a compact component of $f^{-1}(y)$, $y \varepsilon \text{int } f(U)$. Here int U means interior relative to the containing plane P. Throughout the paper interior will be used in that sense. Similarly closure, abbreviated cl, will mean closure relative to the containing plane. Similarly open and closed, unless otherwise stated, will mean relative to the plane.

If the restriction that $m(X)$ contain interior points in 1.1 or the restriction that X is simply connected is removed in 1.2, examples can be constructed wherein the conclusion does not hold.

Closely related to the notion of stability is the notion of an *essential maximal model continuum* studied extensively by T. Radó, P. V. Reichelderfer, and H. Federer. For example, see [5], [6], [1], and [2]. The following is obtained.

1.3. Theorem. *Let $f : X \rightarrow P$ be a compact mapping defined on a simply*

Received May 5, 1961. This research was supported by National Science Foundation grant G-7343 at Georgia Institute of Technology.