PERIODIC AUTOMORPHISMS OF GROUPS AND KNOTS

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1. **Introduction.** By a (topological) rotation of the 3-sphere S^3 we mean an orientation-preserving homeomorphism $R : S^3 \to S^3$ whose set of fixed points is homeomorphic to a circle. R has period q if R^q is the identity and no smaller power of R is the identity. A knot K in S^3 has q as a period if there exists a rotation R of period q with $R(K) = K$ which leaves no point of K fixed.

In this paper we obtain results restricting the possible periods of knots K satisfying

(1.1) K is tame and non-trivial and the commutator subgroup of $\pi_1(S^3 - K)$ is free.

In particular the following is proved.

(1.2) Theorem. *Suppose K satisfies* (1.1) *and that its Alexander polynomial $\Delta(t)$ has no repeated roots. If q is a period for K, then the splitting field of $\Delta(t)$ contains the q-th roots of unity.*

Our arguments are entirely group-theoretic, and the topological conclusion depends on a result of Conner [1]. Observe that if q is a period for K, the associated rotation R gives a homeomorphism of $S^3 - K$ onto itself which induces a periodic automorphism R_* of $\pi_1(S^3 - K)$. (Some fixed point of R is taken as base-point for the fundamental group.) Since $S^3 - K$ is an aspherical 3-manifold and R is orientation-preserving, we can apply Theorem (4.2) of [1] to conclude

(1.3) The subgroup of $\pi_1(S^3 - K)$ consisting of elements left fixed under R_* is either trivial or free cyclic.

(Conner states his theorem only for the case where R has period 2. As he remarks, however, his arguments in fact apply to rotations of arbitrary period.) We use this to show that R_* is non-trivial (if K is non-trivial), and our problem reduces to finding restrictions on the possible periods of automorphisms of $\pi_1(S^3 - K)$.

In §2 we establish a result about periodic automorphisms of a certain class of groups which includes all free groups. §3 deals with periodic automorphisms of certain knot groups, and §4 contains discussion of specific examples.

I wish to acknowledge the influence of conversations with P. E. Conner, R. H. Fox, and R. H. Crowell in connection with this paper.

Received February 1, 1961. This research was sponsored by the National Science Foundation through a program administered by Dartmouth College.