SOME GENERATING FUNCTIONS OF WEISNER

By L. Carlitz

1. Weisner [7] has obtained the following expansions:

$$(1) \qquad (1-w)^{\alpha+\beta-\gamma}(1+(x-1)w)^{-\alpha}(1+(y-1)w)^{-\beta}F(\alpha,\beta;\gamma;\zeta) = \sum_{n=0}^{\infty}\frac{(\gamma)_n w^n}{n!}F(-n,\alpha;\gamma;x)F(-n,\beta;\gamma;y),$$

where

$$(2) \qquad \zeta = \frac{xyw}{(1+(x-1)w)(1+(y-1)w)},$$

and

$$(3) \qquad (1-w)^{\alpha-\gamma-1}(1+(x-1)w)^{-\alpha}\exp\left(\frac{-yw}{1-w}\right) \cdot {}_1F_1\left(\alpha;\gamma-1;\frac{xyw}{(1-w)(1+(x-1)w)}\right) = \sum_{n=0}^{\infty} w^n F(-n,\alpha;\gamma-1;x)L_n^{(\gamma)}(y),$$

where $L_n^{(\gamma)}(x)$ is the Laguerre polynomial,

$$(4) \qquad L_n^{(\gamma)}(x) = \frac{(\gamma-1)_n}{n!}\,{}_1F_1(-n;\gamma+1;x).$$

For another proof of (3) see Rainville [5; 213]; see also Brafman [2].

It may be of interest to point out that (1) can be proved in a straightforward way as follows. Since

$$(1+(x-1)w)^{-\alpha} = ((1-w)+xw)^{-\alpha} = (1-w)^{-\alpha}\left(1+\frac{xw}{1-w}\right)^{-\alpha},$$

we have by (1) and (2)

$$(5) \qquad (1-w)^{\alpha+\beta-\gamma}(1+(x-1)w)^{-\alpha}(1+(y-1)w)^{-\beta}F(\alpha,\beta;\gamma;\zeta) = (1-w)^{\alpha+\beta-\gamma}\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{n!\,(\gamma)_n}\frac{(xyw)^n}{(1+(x-1)w)^{n+\alpha}(1+(y-1)w)^{n+\beta}} = \sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{n!\,(\gamma)_n}\frac{(xyw)^n}{(1-w)^{\gamma+2n}}$$

Received April 24, 1961. Supported in part by National Science Foundation grant G-9425.