EXPANSIONS IN SERIES OF HOMOGENEOUS POLYNOMIAL SOLUTIONS OF THE TWO-DIMENSIONAL WAVE EQUATION

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1. **Introduction.** Let $P_n(x, y)$ be a homogeneous polynomial of degree n. We shall consider series expansions of the form

$$u(x, y) = \sum_{n=0}^{\infty} P_n(x, y). \tag{1.1}$$

It is a familiar fact, M. Bôcher [1], that if the $P_n(x, y)$ are harmonic functions, then the series (1.1) converges in a circle $x^2 + y^2 < \rho^2$ and not throughout any larger including domain. Examples show that it may also converge on certain diameters of the circle extended beyond its circumference. In a paper by P. C. Rosenbloom and the author [2] it is shown that if $P_n(x, \sqrt{t})$ is a polynomial solution of the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},$$

then series (1.1) converges in an infinite strip $|t| < \rho$ and not throughout any larger including continuum. It may also converge on portions of the t-axis outside the strip. In the present note it is proposed to study the series (1.1) when the $P_n(x, y)$ are solutions of the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}. \tag{1.2}$$

It will be shown that the region of convergence is then a rectangle with sides that are characteristics, $|x + y| < \rho_1$, $|x - y| < \rho_2$, plus perhaps portions of the diagonals of the rectangle extended beyond its interior. We shall also show that if $u(x, y)$ is the sum of the series, then the double series Taylor development of $u(x, y)$ about the origin has for its region of convergence a square $|x| + |y| < \rho = \min(\rho_1, \rho_2)$, plus perhaps portions of the diagonals of the square extended beyond its interior.

2. **The homogeneous polynomial solutions.** Set

$$P_n(x, y) = \sum_{k=0}^{n} c_k y^k x^{n-k},$$

and determine the constants c_k to make $P_n(x, y)$ a solution of (1.2). These

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