DEFINITE DIVERGENCE OF THE CONJUGATE FOURIER SERIES

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1. Let

$$\sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx) \tag{1.1}$$

be the conjugate series of the Fourier series corresponding to the function $f(x)$ which is integrable in the sense of Lebesgue over the interval $(-\pi, \pi)$ and is defined outside this interval by periodicity. The conjugate function associated with the above conjugate series is

$$g(x) = \frac{1}{2\pi} \lim_{\epsilon \to 0} \int_{\epsilon}^{\pi} \psi(t) \cot \frac{t}{2}\, dt, \tag{1.2}$$

where

$$\psi(t) = f(x + t) - f(x - t).$$

Prasad [3] has shown that if at a point x, the integral (1.2) diverges to $+\infty$ or to $-\infty$, the Abel limit of (1.1) will also diverge to the same value.

Moursund [2] has proved that if at a point x,

$$\int_0^t | \psi(u) |\, du = O(t),$$

the divergence of $g(x)$ to $+\infty(-\infty)$ is a necessary and sufficient condition for the divergence of (1.1) to $+\infty(-\infty)$ when summed by Riesz's equivalent of the Cesàro method (C, δ) with $\delta > 0$.

Anderson [1] has shown that at a point x where

$$\int_{\epsilon}^{\delta} \left| \frac{\psi(t)}{t} - \frac{\psi(t + 2\epsilon)}{t + 2\epsilon} \right| dt = O(1),$$

as $\epsilon \to 0$, where δ is a positive constant, the definite divergence to $+\infty(-\infty)$ of the integral (1.2) is a necessary and sufficient condition for the definite divergence of the series (1.1) to $+\infty(-\infty)$. The object of this note is to prove the following theorem:

Theorem. *At a point x where*

$$\Psi(t) = \int_0^t \psi(u)\, du = O(t) \tag{1.3}$$

$$\int_{\epsilon}^{\delta} \frac{| \psi(t + \epsilon) - \psi(t) |}{t}\, dt = O(1) \tag{1.4}$$

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