n-CYCLIC ELEMENTS. I

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Cyclic elements of higher order were introduced by G. T. Whyburn [7] and [8] following a suggestion of R. L. Wilder. An historical account will be found in Whyburn's expository paper [9]. The original approach to this notion was based on the Vietoris (mod 2) homology theory and, in consequence, was valid for compact metric spaces. Moreover, the results obtained by Whyburn concerned the ranks of the homology groups thus excluding any consideration of torsion.

In this note the basic space is assumed to be compact Hausdorff, and the Alexander-Kolmogoroff cohomology theory, with random coefficient group, will be used.

In §§2 and 3 we exhibit some of the basic properties of cyclic elements and their use in determining the cohomology structure of a space. Paragraph 4 concerns itself with an application to the co-dimension theory of H. Cohen [2].

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1. **Preliminaries.** Throughout this note let $H^n(X, A)$ be the n-th dimensional Alexander-Kolmogoroff cohomology group, as set forth in Spanier [5] of the space X modulo a closed subset A. When $A = \square$ (= the empty set), we write $H^n(X)$. The coefficient group is taken to be any non-trivial additive Abelian group.

The following well known concepts and theorems for a compact Hausdorff space X, with closed subsets A and B, are stated without proofs and will be used without specific reference. (For detailed proofs see [6].)

Notation. If $A \subset B$ and $h \in H^n(B)$ and the homomorphism $i^*: H^n(B) \to H^n(A)$ is induced by the injection $i: A \subset B$, then $i^*(h)$ will be denoted by $h \mid A$.

1.1. DEFINITION. Let $A \subset B$ and let $h \in H^n(A)$. Then h is extendable to B if there is an $r \in H^n(B)$ such that $r \mid A = h$.

1.2 THEOREM. *Let $h \in H^n(A)$. If h is not extendable to X, then there is a closed subset R of X such that* (1) *h is not extendable to $R \cup A$ and* (2) *if R_0 is any proper closed subset of R, then h is extendable to $R_0 \cup A$.*

Such a set R is called a roof for h in X.

A "dual" to 1.2 is

1.3 THEOREM. *Let $h \in H^n(A)$ and $h \neq 0$. Then there is a closed set F in A such that* (1) *$h \mid F \neq 0$ and* (2) *if F_0 is any closed proper part of F, then $h \mid F_0 = 0$.*

The set F is called a floor for h in A.

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