ON THE COEFFICIENT GROUP IN COHOMOLOGY

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1. **Introduction.** In this paper we intend to examine the effect of the use of various coefficient groups in the cohomology theory of compact Hausdorff spaces. The results obtained are valid in any cohomology theory satisfying the basic axioms given by Eilenberg and Steenrod [4] and satisfying the continuity axiom. In particular the results apply within the theories of Čech and Alexander-Kolmogoroff-Wallace. A variant of the latter method of construction is used in all procedures which follow.

The principal methods used here are threefold. First, the effect of homomorphic transformations of the coefficient group are examined, an exact sequence being the principal result. Secondly, it is proven that if the coefficient group is a direct product, then the cohomology groups of a compact space decompose naturally as a direct product. This is then extended to a more comprehensive result on direct limits. The decompositions obtained are invariantly defined, in the sense that at no point is it necessary to select a basis for any group. This has the effect of making all homomorphisms natural, yielding not only information about the group, but producing simple decompositions for the coboundary homomorphisms and the homomorphisms induced by continuous mappings. The remainder of the paper is concerned with applications.

2. **Finitely-valued cohomology groups.** A variant of the Alexander-Kolmogoroff-Wallace groups of a pair (X, A), where X is a topological space and A is a subset of X may be rapidly defined as follows. For an Abelian group G, denote by $F^p(X; G)$ the group of all functions (p-cochains) from X^{p+1} (the Cartesian product of X with itself $p + 1$ times) to G which take on *at most a finite number of values* in G, the group operation defined by pointwise addition. A homomorphism $\bar{\delta} : F^p(X; G) \to F^{p+1}(X; G)$ is defined, for $\phi \in F^p(X, G)$ and $(x_0, \cdots, x_{p+1}) \in X^{p+2}$, by

$$(\bar{\delta}\phi(x_0, \cdots, x_{p+1}) = \sum_{i=0}^{p+1} (-1)^i \phi(x_0, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{p+1}).$$

It follows that $\bar{\delta}\,\bar{\delta} = 0$.

For any collection $\mathfrak{U}$ of subsets of X, write $\mathfrak{U}^{(p+1)} = \cup \{U^{p+1} \mid U \in \mathfrak{U}\}$. In particular, if $\mathfrak{U}$ is an open covering of X, then $\mathfrak{U}^{(p+1)}$ is a neighborhood of the diagonal of X^{p+1}; conversely, any neighborhood of the diagonal of X^{p+1} contains $\mathfrak{U}^{(p+1)}$ for some open covering $\mathfrak{U}$ of X.

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