EFFECT OF THE ADDITION OF THE UNIT SET ON THE ORDER OF A SIMPLE MONIC SET OF POLYNOMIALS

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Given a simple monic set $\{p_n(z)\}$ of order ω, the question is whether the addition of the unit set $\{z^n\}$ affects or does not affect the order ω. (For definitions and notation, see articles 1 and 2 of [1].) It might appear, at first glance, that the addition of the unit set affects the coefficients p_{ij} so slightly that no change of the order may be expected. This is not really the case. We give here a theorem which shows precisely to what extent the order of an algebraic simple monic set may be changed by the addition of the unit set. From the theorem it will appear clear that, given any positive integer N however large we can always construct a simple monic set $\{p_n(z)\}$ of order ω, but $\{p_n(z)\} + \{z^n\}$ is of order $N\omega$ or ω/N.

THEOREM. *If $\{p_n(z)\}$ is an algebraic simple monic set of degree m and order ω, then the sum set*

$$\{u_n(z)\} = a\{p_n(z)\} + b\{z^n\}, \qquad a + b = 1, \tag{1}$$

is of order Ω such that

$$\omega/(m - 1) \leq \Omega \leq (m - 1)\omega. \tag{2}$$

The upper and lower bounds are both attainable.

Since $\{p_n(z)\}$ is an algebraic simple monic set of degree m, then [1; 527]

$$(P - I)^m = 0, \tag{3}$$

where P is the matrix of coefficients associated with the set $\{p_n(z)\}$. This equation may be written as

$$P^m + a_1P^{m-1} + a_2P^{m-2} + \cdots + a_mI = 0, \tag{4}$$

from which

$$P^{m-1} + a_1P^{m-2} + a_2P^{m-3} + \cdots + a_{m-1}I + a_m\prod = 0, \tag{5}$$

where $\prod = P^{-1}$ is the matrix of operators associated with the set $\{p_n(z)\}$.

Since the set $\{p_n(z)\}$ is of order ω, then

$$| p_{ni} | \leq O\{n^{(\omega+\epsilon)n}\} \qquad (0 \leq i \leq n - 1), \tag{6}$$

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