CLASSES OF SOLUTIONS OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS IN THREE VARIABLES

By Stefan Bergman

1. **Introduction.** In a series of papers, see [1]—[11] and [18]—[26], a general method of using functional operators to study the properties of functions satisfying a linear partial differential equation has been developed.

Remark 1.1. The present paper does not presuppose the knowledge of previous publications.

Stated in the most general terms the idea of the method is to find an operator P which acts on a function f belonging to a given class $\mathcal{A}$ and which transforms it into a solution U of a given partial differential equation $L(U) = 0$. If the class of solutions is denoted by $\mathcal{S}$, then we may write symbolically

$$U = P(f) \qquad (f \in \mathcal{A},\ U \in \mathcal{S}); \tag{1.1}$$

f is called the associate function of U with respect to the operator P.

In many instances it is possible to infer many of the properties of the functions of class $\mathcal{S}$ from a knowledge of the behaviour of functions of the class $\mathcal{A}$.

Remark 1.2. In order to prevent any misunderstanding it should be stressed that the functions f and U are considered *not* in some fixed region, but in the whole domain of regularity of the individual function.

As a trivial example of this method, consider the case where L is the two-dimensional Laplacian, $\nabla^2 U = 0$; $\mathcal{S}$ is the class of harmonic functions of two variables, and $\mathcal{A}$ is the class of all analytic functions of one complex variable. In this case the operator P consists merely in taking the real part of f, i.e.,

$$U = \mathrm{Re}\,[f] \qquad (f \in \mathcal{A},\ U \in \mathcal{S}). \tag{1.2}$$

Obviously the properties of solutions U can be deduced from a knowledge of the behaviour of the associate function f, and, conversely, a given solution U determines a unique associate function f (to within an additive constant).

When more complicated differential equations are studied, the operators become more complicated and more difficult to discover. It has been possible to find operators in the form of definite integrals which can produce complex solutions of linear partial differential equations (*in two variables*) of both hyperbolic and elliptic types from functions of a single real variable and complex variable, respectively. See [4], [5], [10; §3]. It is then sufficient to take the real part of such a complex solution to obtain a real solution of the differential equation. Among others there exist operators of such nature that relations between the properties of the associate function and the corresponding complex solution can be found with relative ease. See [7; §5–§7, 150–152]. It has,

Received December 17, 1945.