AN INVOLUTORIAL SPACE TRANSFORMATION ASSOCIATED WITH A $Q_{1,n}$ CONGRUENCE

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1. **Introduction.** Recently the present author [2] gave an analytical discussion of a non-involutorial transformation associated with the congruence of lines on a plane curve of order n having an $(n - 1)$-point and a secant through that point. In what follows an involutorial transformation associated with such a congruence is studied, the procedure again being almost entirely analytical.

DePaolis [1] some years ago published a synthetic discussion of four particular involutorial transformations associated with the congruence of lines which are bisecants of a space curve of order μ and a chord meeting the curve in $\mu - 1$ points. Of these transformations the "second species" may be considered to be, in a sense, the most general since for a space curve of given order and for given multiplicities of this curve and of the bisecant upon the pointwise invariant surface the order of this transformation is higher than those of the other three types. The present transformation is equivalent to this "second species" from the standpoint of resemblance in those properties which DePaolis described. No essential properties of the transformation have been lost through the use of a plane curve while the analytical treatment gives many details not mentioned in [1] (such as the existence of parasitic lines) and points out several errors, the order given for his curve Γ being a case in point.

The bundle of lines through the multiple point is not considered as belonging to the congruence and the tangents to the curve at this point are first assumed to be distinct.

Given the plane n-ic r, a line s meeting r at an $(n - 1)$-point A, and a pencil of surfaces

$$| F_m | : s^{m-2} g_{4m-4}$$

which, as indicated, are of order m and contain s $(m - 2)$-times. Through a generic point $P(y)$ there passes a single F of $| F |$. The unique line t belonging to the congruence and passing through $P(y)$ meets F a second time in one residual point $Q(x)$, image of $P(y)$ under the transformation thus defined. The residual base curve of $| F |$ has been denoted by g, is of order $4m - 4$, and is considered non-composite. (This curve corresponds to DePaolis' curve Γ whose order seems to be incorrectly given). It will be shown that r, s and g are fundamental curves of the involution and that A is a fundamental point of the second kind.

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