FIBERINGS WITH SINGULARITIES

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1. **Introduction.** The concept of fiber mapping is well known in topology and includes the two slightly different concepts of fiber space [5] and fiber bundles [9; 10]. The original treatment of fiberings goes back to Seifert [7] who permitted a certain type of singularity. We study here a different type of singularity which may be roughly described by saying that certain fibers are pinched to points. A simple example is given by the circles of latitude and the north and south poles of a two-sphere. It is natural to consider singularities of this type because of their analogy with stationary points under compact groups of transformations. Our own interest arose partly in this way and partly through an attempt to verify the following conjecture: Euclidean space cannot be fibered by a compact fiber. Incidentally we treat this conjecture for a few special cases in some concluding remarks.

2. **Definitions.** Let R, F, and B be polyhedra, let L be a closed subset of R, and let P be a continuous mapping of R onto B satisfying the following conditions:

(1) P is an open mapping,

(2) P is topological on L,

(3) $R - L$ is a fiber bundle, with P as projection, over $B - P(L)$, with fiber F. Then P is called a singular fiber mapping and the points of L are called the singularities. For the definition of fiber bundle used here, see [9].

We confine our attention to differentiable singular fiber mappings which in addition to (1), (2), and (3) are also required to satisfy the following conditions (4) and (5).

(4) $R - L$, $B - P(L)$, and F are connected differentiable manifolds; F is compact.

(5) The group associated with F consists of differentiable homeomorphisms and all mappings occurring in the definition of (3) are differentiable.

The word differentiable is here used to imply the existence of continuous first partial derivatives. Conditions (4) and (5) guarantee that the non-singular fibers are imbedded differentiably and also depend differentiably on $B - P(L)$.

The following theorem proved by Steenrod [9] will be useful: If A is a compact fiber bundle over B and B is an ANR, then A is a fiber space over B. This shows that the covering homotopy theorem is applicable.

In analogy with the situation for groups of transformations we shall call a subset of R invariant provided that whenever it contains a point it contains

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