THE SUPER-ABSOLUTE CESÀRO SUMMABILITY OF FOURIER SERIES

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1. **Introduction.** We suppose throughout in this note that $f(x)$ is integrable in the sense of Lebesgue and periodic with period 2π, and that the Fourier series of $f(x)$ is $\sum c_m e^{imx}$. Let

$$\phi(t) = \phi_x(t) = \tfrac{1}{2}\{f(x+t) + f(x-t)\}.$$

We denote the mean function of order α of $\phi_x(t)$ by

$$[\phi_x(t)]_\alpha = \frac{\alpha}{t^\alpha}\int_0^t (t-u)^{\alpha-1}\phi_x(u)\,du \qquad (0 < t \leq \pi, \alpha > 0).$$

Further, we define

$$[\phi_x(t)]_\alpha = [\phi_x(-t)]_\alpha \quad \text{and} \quad [\phi_x(t)]_\alpha = [\phi_x(t+2\pi)]_\alpha$$

for any t not congruent to zero with modulus 2π.

Write

$$\Delta_k f = \Delta_k f(\theta, h) = \sum_{\nu=0}^{k} (-1)^\nu \binom{\nu}{k} f(\theta + kh - 2\nu h) \qquad (k = 1, 2, \cdots),$$

and

$$M_p(\Delta_k f) = \left\{\frac{1}{2\pi}\int_0^{2\pi} |\Delta_k f(\theta, h)|^p \, d\theta\right\}^{1/p}.$$

In an earlier paper [5], I have proved that if

$$M_p^p(\Delta_1 f) = O\left(h\left(\log\frac{1}{h}\right)^{-1-\alpha p}\right) \qquad (h \to 0, \alpha > 0, 2 \geq p > 1), \tag{1.1}$$

then the series

$$\sum_{-\infty}^{\infty}{}' \, |c_m| (\log |m|)^T$$

converges for $T < \alpha + p^{-1} - 1$, where $\sum'$ denotes the summation for $|m| > 1$.

In the case $\alpha + p^{-1} - 1 > 0$, the conclusion implies the absolute convergence of $\sum c_m$, which can also be deduced from a theorem of Szász [9].

Now let $l = \{l_n\}$ be an increasing sequence such that

$$\lim_{n\to\infty} l_n = +\infty.$$

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